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NATURAL, FORCED AND MIXED CONVECTION IN FIBROUS INSULATION

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NATURAL, FORCED AND MIXED CONVECTION IN FIBROUS INSULATION

Aydin Konuk

ABSTRACT

A numerical solution of flow and temperature distribution in fibrous insulation has been obtained. Rectangular and cylindrical geometries have been modeled. Boundary conditions included permeable hot wall and convective heat transfer at the walls. Good agreement has been obtained with published experimental and numerical values on rectangular cavities. The computed velocity and temperature profiles gave a better understanding of flow and heat transfer phenomena in fibrous insulation. Local cold wall and average Nusselt numbers presented provide useful information in the design of the fibrous insulation for concrete reactor vessel and primary coolant piping of the gas cooled nuclear power plants. Average Nusselt number has been correlated with Rayleigh number when only natural convection is present, and with Rayleigh and Reynolds numbers when a combination of natural and forced convection is present.

1 – INTRODUCTION

Extensive theoretical and experimental investigations have been conducted on fibrous type thermal insulation of nuclear reac or during the last decade[5,3,4,1]. These studies were concerned with rectangular geometries of interest in the nuclear reactors. Investigation of the fibrous insulation in cylindrical geometries however does not appear in the literature, except one experimental work[2].

The cylindrical geometry, among others, applies to double-pipe systems, which are widely used to carry hot fluids under high pressure[6]. The inner pipe, subject to high temperature, does not carry the pressure load since it is either perforated or made of non-sealed pieces. The outer wall works at low temperatures because of the temperature drop in the insulation layer and carries the pressure load. Thus, the insulation layer is subjected to large temperature differences across its thickness and to pressure gradients along its hot face. As a result, natural convection and forced convection permeation flows can develop within the insulating layer. These flows can degrade its performance and create hot local spots. This report describes the development and application of a computer program to predict the steady-state performance of fibrous insulation enclosed between two concentric cylinders (horizontal or vertical) or in a rectangular cavity.

Experiments[5,3] show that the flow in fibrous insulation obeys Darcy's Law. For such materials, classified as porous media, the momentum equation is simplified since the pressure gradient is a linear function of the velocity.

It is also assumed that solid and the gas have the same local temperatures. In the present development, except those specified above (and commun to all previous work), no other assumption is made. The fluid properties are allowed to vary locally with temperature, using empirical equations to predict them. In particular the local derivatives of the density on the right hand side of the momentum equation are evaluated directly, rather than being converted into temperature gradients by taking the compressibility factor Z of the gas as Z = 1. Thus, no inaccuracies are introduced when the gas departs largely from ideal gas behaviour.

Various boundary conditions are included in the model. Except for the horizontal cylinder, all three geometries considered include, on both the inner and the outer walls, convective boundary conditions or specified wall temperatures, and external flow over a perforated inner surface with its own permeability. The last boundary condition for a horizontal cylinder requires a three dimensional solution, therefore it has been excluded. The derivation of the equations and boundary conditions is described in the second section of the report.

A numerical method has been developed for the solution of the two non-linear partial differential equations (momentum and energy equations) with the proper boundary conditions resulting from the present formulation. Except at high Rayleigh or Reynolds numbers, no convergence problems have been encountered. The numerical procedure is the subject of the third section.

The computed results are in the forth section. Velocity and temperature fields as well as local and average Nusselt numbers are presented.

Conclusions and remarks concerning future analytical and experimental work are in the final section.

Appendix contains a listing of the computer program with input information.

2 – FORMULATION OF EQUATIONS AND BOUNDARY CONDITIONS

2.1 – Horizontal Cylinder

The equations and boundary conditions describing flow and heat transfer in the case of a horizontal cylinder are formulated considering the half-circular cross section illustrated in Figure 1.

Considerations of symmetry allow the modeling of one half of the cross section instead of the full cross section. This has the advantage of decreasing the computer execution time and memory requirements. Equations of conservation of mass, momentum and energy are written for the two dimensional problem, the independent variables beeing the angular direction θ and the radial direction r. The axial direction is not considered assuming that flow and temperature fields do not change in that direction. This is true there is no permeation flow through the inside wall. A three dimensional modeling of the horizontal cylinder has not been possible within the framework of the a present numerical method of solution due to the large increase in the computer execution time and memory requirements.

Conservation of Mass

The equation of conservation of mass is:

$$\frac{\partial}{\partial r}(r \rho v_r) + \frac{\partial}{\partial \theta}(\rho v_\theta) = 0 \tag{1}$$

Conservation of Momentum

For porous media, equation of conservation of momentum reduces to Darcy's law. In two dimensional cylindrical coordinates with r and θ, Darcy's law is written as

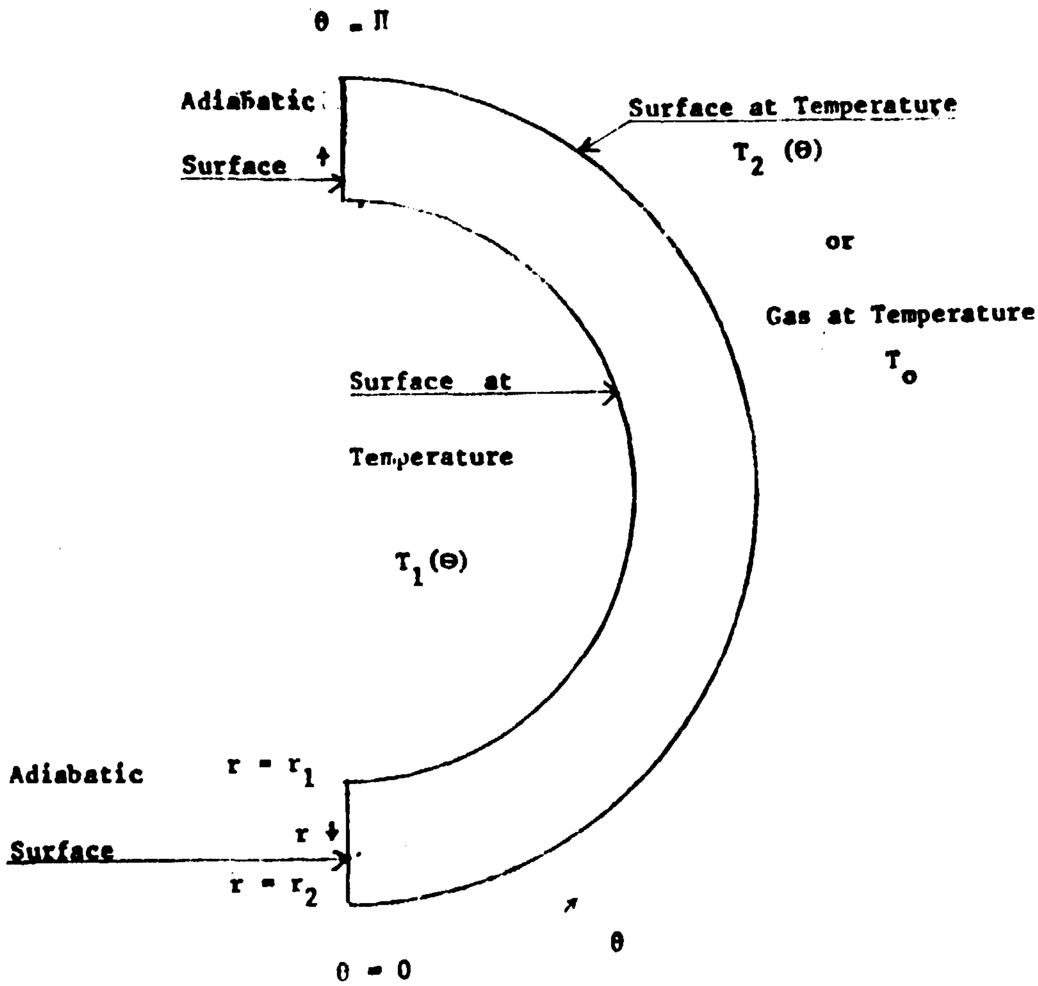


Figure 1 – Horizontal Cylinder Geometry for which the boundary conditions are described

$$v_r = \frac{K_r}{\mu} (\rho g_r - \frac{\partial p}{\partial r}) \tag{2}$$

for the r — momentum equation and as

$$v_\theta = \frac{K_\theta}{\mu} (\rho g_\theta - \frac{\partial p}{r\partial \theta}) \tag{3}$$

for the θ -- momentum equation. The velocity v in these equations is the superficiel velocity (volume flow rate of flow through a unit cross section area of the fluid plus solid) averaged over a small region of space-small with respect to the macroscopic dimensions of the flow system but large with respect to the pore size.

Equations (2) and (3) are combined to eliminate the pressure, wich gives

$$\frac{1}{K_r} \frac{\partial}{\partial \theta} (\mu v_r) - \frac{1}{K_\theta} \frac{\partial}{\partial r} (\mu r v_\theta) = \frac{\partial}{\partial \Theta} (\rho g_r) - g_\theta \frac{\partial}{\partial r} (r \rho) \tag{4}$$

To reduce the 2 velocities into one variable and to eliminate the equation of conservation of mass, a stream function (which satisfies the equation of continuity) is defined by

$$v_r = \frac{1}{r\rho} \frac{\partial \psi}{\partial \theta} \tag{5}$$

$$v_\theta = - \frac{1}{\rho} \frac{\partial \psi}{\partial r} \tag{6}$$

With the above definitions of the stream function, and with $g_r = g\cos\theta$, $g_\theta = -g\sin\theta$, the equation of conservation of momentum becomes

$$\frac{1}{rK_r} \frac{\partial}{\partial \theta} (\nu \frac{\partial \psi}{\partial \theta}) + \frac{1}{K_\theta} \frac{\partial}{\partial r} (r\nu \frac{\partial \psi}{\partial r}) = g(\cos\theta \frac{\partial \rho}{\partial \theta} + r\sin\theta \frac{\partial \rho}{\partial r}) \tag{7}$$

Conservation of Energy

The equation of conservation of energy is:

$$\frac{\partial}{\partial r} (r\rho c_p v_r T) + \frac{\partial}{\partial \theta} (\rho c_p v_\theta T) - \frac{\partial}{\partial r} (r \lambda \frac{\partial T}{\partial r}) - \frac{1}{r} \frac{\partial}{\partial \theta} (\lambda \frac{\partial T}{\partial \theta}) = 0 \tag{8}$$

Using the definitions of the stream function and using the equation of conservation of mass, one obtains:

$$\frac{\partial \psi}{\partial \theta} \frac{\partial}{\partial r} (c_p T) - \frac{\partial \psi}{\partial r} \frac{\partial}{\partial \theta} (c_p T) - \frac{\partial}{\partial r} (r\lambda \frac{\partial T}{\partial r}) - \frac{1}{r} \frac{\partial}{\partial \theta} (\lambda \frac{\partial T}{\partial \theta}) = 0 \qquad (9)$$

Thus, the original group of governing equations has been reduced to two equations with two dependent variables ψ and T. The coefficients ρ, ν, c_p, and λ are variable coefficients which are functions of T and the system pressure.

Boundary Conditions

Since the governing differentiel equations (7) and (9) are each of second order in ψ and T, these equations are subject to two boundary conditions on the stream function and two on the temperature. For the problem shown in Figure 1, the boundary conditions are:

At $\theta = 0$ and $\theta = \Pi$

$\psi = 0$ ($v_\theta = 0$ because of the symmetry)

$\frac{\partial T}{\partial \theta} = 0$ (no heat flux because of symmetry)

At $r = r_1$; there are two possible boundary conditions:

(a) $T = T_1(\theta)$

(b) $h_1 (T_g - T) = -\lambda \frac{\partial T}{\partial r}$

Condition (a) corresponds to a given inside wall temperature. Condition (b) is a convective boundary condition. T_g is the bulk temperature of the gas flowing through the inner pipe, and h_1 is the heat transfer coefficient. The temperature drop across the wall is neglected. For high values of h, the temperature drop $(T_g - T)$ will be small; in this case condition (b) is equivalent to condition (a), with $T = T_1(\theta) = T_g$.

At $r = r_2$, again two possible boundary conditions are considered:

(a) $T = T_2(\theta)$

(b) $h_2 (T - T_0) = -\lambda \frac{\partial T}{\partial r}$

Condition (a) corresponds to a given outside wall temperature. Condition (b) is again a convective boundary condition. This boundary condition is valid when there is no outside insulation. T_0 is the temperature of the gas outside (usually atmospheric air) and h_2 is the heat transfer coefficient (usually natural convection heat transfer coefficient for a horizontal cylinder). Condition (b) allows for the angular variation of the outside wall temperature.

2.2 – Vertical Cylinder

The equations and boundary conditions for flow and heat transfer for insulation packed between two vertical cylinders shown in Figure 2 are written in cylindrical coordinates r and x. For this geometry, velocity and temperature fields do not change with the angular coordinate θ, therefore a two dimensional modeling with the radial and axial coordinates is sufficient.

Conservation of Mass

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{\partial}{\partial x}(\rho v_x) = 0 \qquad (10)$$

Conservation of Momentum

Darcy's law, with velocity components v_r and v_x, is:

$$v_r = -\frac{K_r}{\mu}\left(\rho g_r - \frac{\partial p}{\partial r}\right) \qquad (11)$$

$$v_x = \frac{K_x}{\mu}\left(\rho g_x - \frac{\partial p}{\partial x}\right) \qquad (12)$$

Eliminating the pressure, and nothing that $g_r = 0$ and $g_x = -g$, one obtains

$$\frac{1}{K_r}\frac{\partial}{\partial x}(\mu v_r) - \frac{1}{K_x}\frac{\partial}{\partial r}(\mu v_x) = g\frac{\partial \rho}{\partial r} \qquad (13)$$

A stream function is defined by

$$v_r = \frac{1}{\rho r}\frac{\partial \psi}{\partial x} \qquad (14)$$

$$v_x = -\frac{1}{\rho r}\frac{\partial \psi}{\partial r} \qquad (15)$$

Combining Equations (13), (14), and (15) one obtains

$$\frac{1}{K_r}\frac{1}{r}\frac{\partial}{\partial x}\left(\nu\frac{\partial \psi}{\partial x}\right) + \frac{1}{K_x}\frac{\partial}{\partial r}\left(\frac{\nu}{r}\frac{\partial \psi}{\partial r}\right) = g\frac{\partial \rho}{\partial r} \qquad (16)$$

Conservation of Energy

The equation of conservation of energy, written in two dimensional cylindrical coordinates with coordinates r and x is:

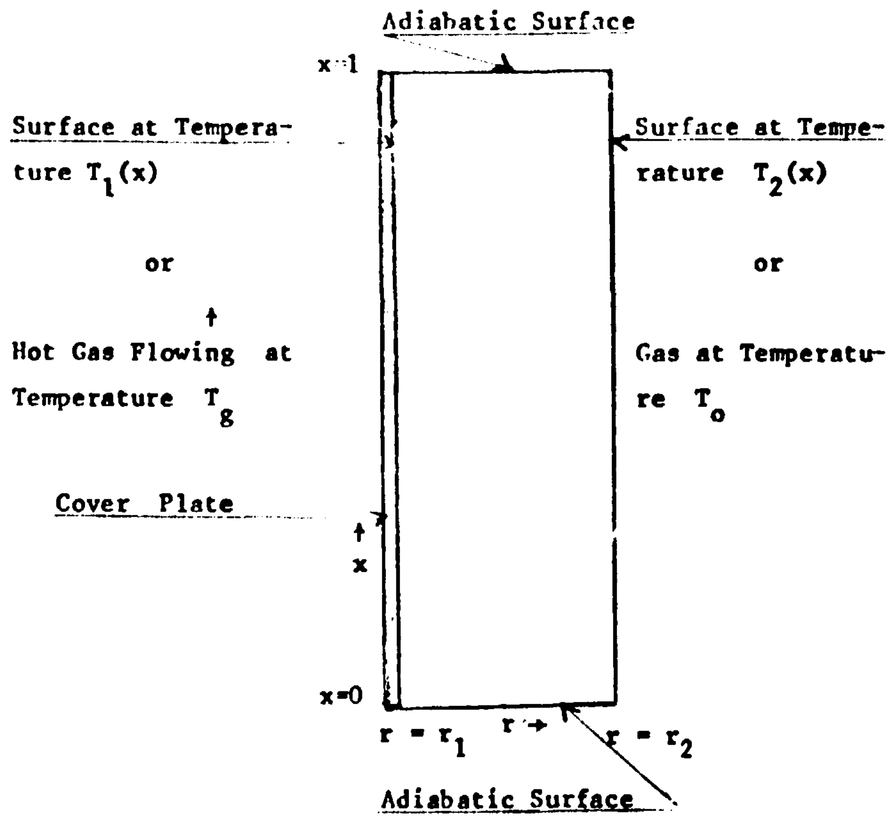


Figure 2 – Vertical Cylinder ieometry for which the boundary conditions are described

$$\frac{\partial}{\partial r}(r\,v_r\rho\,c_p\,T) + \frac{\partial}{\partial x}(r\,v_x\rho\,c_p\,T) - \frac{\partial}{\partial r}(r\,\lambda\,\frac{\partial T}{\partial r}) - r\,\frac{\partial}{\partial x}(\lambda\,\frac{\partial T}{\partial x}) = 0 \qquad (17)$$

Introducing the stream function and using the equation of conservation of mass, one obtains

$$\frac{\partial\psi}{\partial x}\,\frac{\partial}{\partial r}(C_pT) - \frac{\partial\psi}{\partial r}\,\frac{\partial}{\partial x}\,(C_pT) - \frac{\partial}{\partial r}\,(r\,\lambda\,\frac{\partial T}{\partial r}) - r\,\frac{\partial}{\partial x}\,(\lambda\,\frac{\partial T}{\partial x}) = 0 \qquad (18)$$

Boundary Conditions

. The boundary conditions for a vertical cylinder include the possibility of permeation flow through the perforated inner wall of a double-pipe system. The cover plate is considered as a porous medium with own permeability K_c, and thickness a. The boundary conditions are:

$x = 0$ and $x = L$;

$\psi = 0$ ($v_x = 0$, solid wall)

$\frac{\partial T}{\partial x} = 0$ (no heat flow through the solid wall)

$r = r_1$;

(a) $\psi = 0$ ($v_r = 0$, solid inner wall without perforations)

or

$$\text{(b)}\quad \frac{\partial\psi}{\partial r} + \frac{aK}{K_c}\,\frac{\partial^2\psi}{\partial x^2} = \frac{rK}{\nu}\,(g\,\rho + \frac{\partial p_o}{\partial x}) \qquad (19)$$

Boundary condition (b) corresponds to permeation flow through the perforated wall, where $\frac{\partial p_o}{\partial x}$ is the pressure gradient in the inner pipe due to the flow of the gas.

There also two possible boundary conditions on T:

(a) $T = T_1(x)$ (given inner wall temperature)

or

(b) $h_1\,(T_g - T) = -\lambda\,\frac{\partial T}{\partial r}$ (convective boundary condition as in the horizontal cylinder)

$r \quad = r_2$

$\psi \quad = 0 \quad (v_r = 0, \quad$ solid outer wall)

(a) $T = T_2\ (x)$ (given outer wall temperature)

or

(b) $h_2\ (T - T_o) = -\lambda \frac{\partial T}{\partial r}$ (convective boundary condition as in the horizontal cylinder)

Equation (19) is obtained as follows:

Darcy's law applied to the cover plate ($r = r_1$) gives

$$v_r = \frac{K_c}{\mu} (\rho g_r - \frac{\partial p}{\partial r}) \tag{20}$$

With $g_r = 0$ and $\frac{\partial p}{\partial r} = \frac{p - p_o}{a}$, Equation (20) gives

$$p = p_o - \frac{a\mu v_r}{K_c} \tag{21}$$

Differentiating Equation (21) with respect to x and using definition of the stream function, one obtains (at $r = r_1$)

$$\frac{\partial p}{\partial x} = \frac{\partial p_o}{\partial x} - \frac{a}{K_c} \frac{\partial}{\partial x} (\frac{-\nu}{r} \frac{\partial \psi}{\partial x}) \tag{22}$$

Writing the x – momentum equation at $r = r_1$ gives

$$v_x = \frac{K}{\mu} (\rho g_x - \frac{\partial p}{\partial x}) \tag{23}$$

With $g_x = -g$ and using the definition of the stream function, one obtains for $\frac{\partial \psi}{\partial r}$

$$\frac{\partial \psi}{\partial x} = \frac{rK}{\nu} (g\rho + \frac{P}{x}) \tag{24}$$

Substituting $\frac{\partial p}{\partial x}$ from Equation (22) into Equation (24), Equation (19) is derived.

2.3 – Rectangular Geometries

The equations and boundary conditions for the flow and heat transfer in insulation packed in a rectangular cavity are written considering the rectangular cross section shown in Figure 3.

Equations are formulated in two dimensional rectangular coordinates x and y, where x is along the cover plate. The equation are the following.

Conservation of Mass

$$\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) = 0 \tag{25}$$

Conservation of Momentum

$$U_x = \frac{K_x}{\mu}(\rho g_x - \frac{\partial p}{\partial x}) \tag{26}$$

$$U_y = \frac{K_y}{\mu}(\rho g_y - \frac{\partial p}{\partial y}) \tag{27}$$

Eliminating the pressure gives

$$\frac{1}{K_x}\frac{\partial}{\partial y}(\mu v_x) - \frac{1}{K_y}\frac{\partial}{\partial x}(\mu v_y) = g_x \frac{\partial \rho}{\partial y} - g_y \frac{\partial \rho}{\partial x} \tag{28}$$

The stream function is defined by

$$v_x = -\frac{1}{\rho}\frac{\partial \psi}{\partial y} \tag{29}$$

$$v_y = \frac{1}{\rho}\frac{\partial \psi}{\partial x} \tag{30}$$

Combining Equations (28), (29), and (30), one obtains

$$\frac{1}{K_x}\frac{\partial}{\partial y}(\nu \frac{\partial \psi}{\partial y}) + \frac{1}{K_y}\frac{\partial}{\partial x}(\nu \frac{\partial \psi}{\partial x}) = g_y \frac{\partial \rho}{\partial x} - g_x \frac{\partial \rho}{\partial y} \tag{31}$$

Conservation of Energy

$$\frac{\partial}{\partial x}(v_x \rho C_p T) + \frac{\partial}{\partial y}(v_y \rho C_p T) - \frac{\partial}{\partial x}(\lambda \frac{\partial T}{\partial x}) - \frac{\partial}{\partial y}(\lambda \frac{\partial T}{\partial y}) = 0 \tag{32}$$

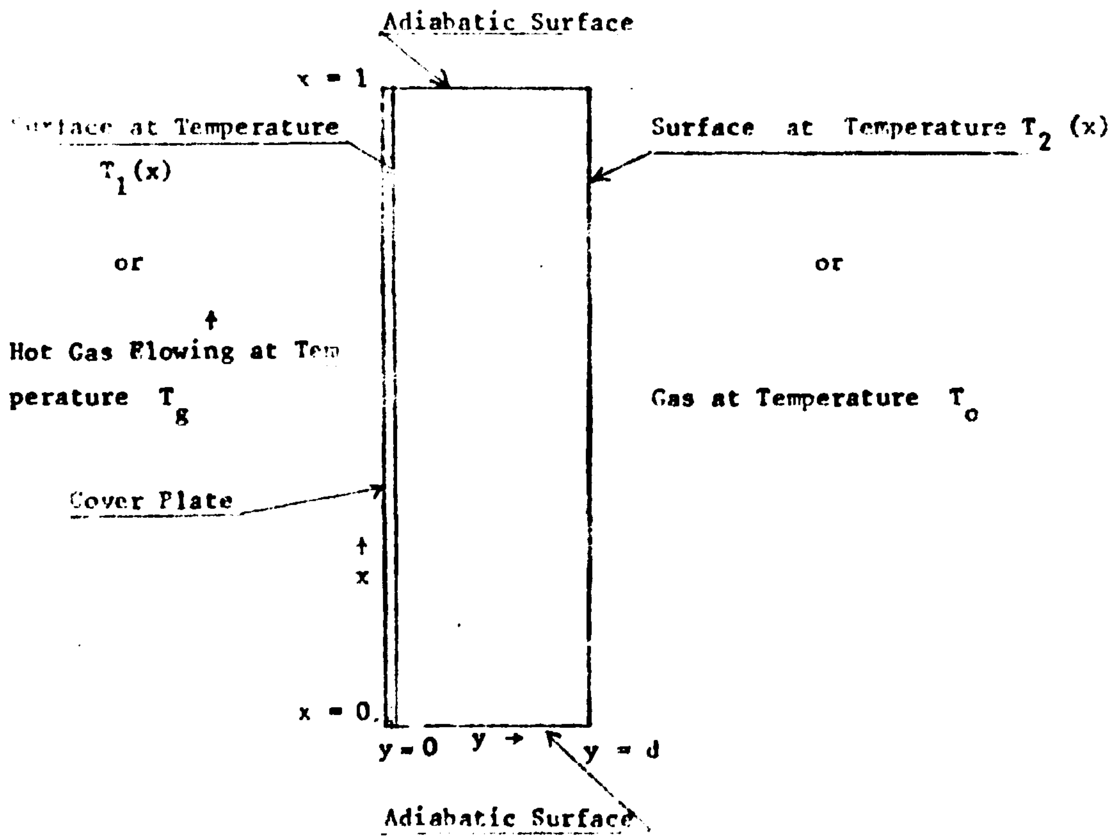


Figure 3 – Rectangular Geometry for which the boundary conditions are described

Combining Equations (25), (26), (27), and (32), results in equation of conservation of energy

$$\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x}(C_p T) - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}(C_p T) - \frac{\partial}{\partial x}(\lambda \frac{\partial T}{\partial x}) - \frac{\partial}{\partial y}(\lambda \frac{\partial T}{\partial y}) = 0 \qquad (33)$$

Boundary Conditions

Boundary conditions are similar to those written for the vertical cylinder.

At $x = 0$ and $x = 1$;

$\psi = 0$ ($v_x = 0$, solid wall)

$\frac{\partial T}{\partial x} = 0$ (no heat flux through the solid wall)

At $y = 0$

$\psi = 0$ ($v_y = 0$, cover plate without perforations)

or

$$\frac{\partial \psi}{\partial y} = -\frac{K}{\nu}(g_x - \frac{\partial p_o}{\partial x}) + \frac{Kag_y}{\nu} - \frac{aK}{K_c}\frac{\partial^2 \psi}{\partial x^2} \qquad (34)$$

$T = T_1(x)$ (given cover plate temperature)

or

$h_1 (T_g - T) = -\frac{\partial T}{\partial y}$ (convective boundary condition)

At $y = 1$

$\psi = 0$ ($v_y = 0$, solid wall)

$T = T_2(x)$ (given outer wall temperature) or

$$h_2 \, (T - T_0) = - k \frac{\partial T}{\partial y} \quad \text{(convective boundary condition)}$$

Equation (34) is derived as follows.

Darcy's law across the cover plate gives

$$\frac{\mu v_y}{K_c} = \rho g_y - \frac{\partial p}{\partial y} \tag{35}$$

or

$$\frac{\mu v_y}{K_c} = \rho g_y + \frac{p_o - p}{a} \tag{36}$$

Differentiating Equation (36) with respect to x and rearranging, one obtains

$$\frac{\partial p}{\partial x} = \frac{\partial p_o}{\partial x} + a \frac{\partial}{\partial x} \left(\rho g_y - \frac{\mu v_y}{K_c} \right) \tag{37}$$

Writing the x-momentum equation at y = 0

$$v_x = \frac{K}{\mu} \left(g_y - \frac{\partial p}{\partial x} \right) . \tag{38}$$

Substituing $\frac{\partial p}{\partial x}$ from Equation (37) into Equation (38) gives the permeation flow boundary condition on ψ.

3 – NUMERICAL SOLUTION

3.1 – Horizontal Cylinder

The coupled Equations (7) and (9), and the boundary condition equations are solved numerically using a finite difference method implemented by a computer program called FINS (Fibrous Insulation). The numerical procedure is described as follows, considering the grid shown in Figure 4

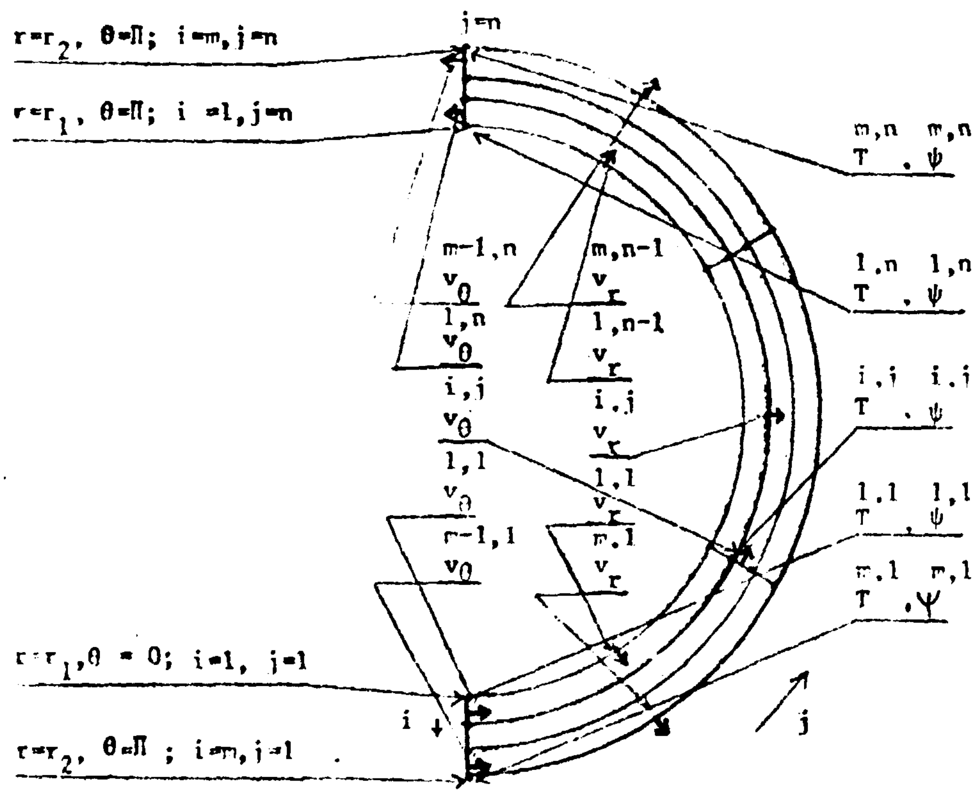


Figure 4 – Horizontal Cylinder, Finite Difference Grid

The derivates terms in ψ and ρ in Equation (7) are expressed as central differences for all the interior points.

Thus, $(n-2)\ (m-2)$ algebraic equations are obtained. Boundary conditions expressing that $\psi^{i,j} = 0$ on the periferial points supply $2(n+m)-4$ algebraic equations, thus yielding a total of $n \times m$ equations, with $n \times m$ stream function $\psi^{i,j}$ variables. When the gas viscosities $\nu^{i,j}$ and densities $\rho^{i,j}$ are known at each grid point, Equation (7), which is a non-linear partial differential equation, is reduced to a system of linear algebraic equations which can be solved readily by numerical methods.

Similarly, Equation (9), which is also a non-linear partial differentiel equation, is put in a finite difference form using central differences for T and ψ derivatives at the interior points. For the boundary conditions, $\frac{\partial T}{\partial \theta}$ is written as a forward difference, and $\frac{\partial T}{\partial r}$ at $r = r_1$ and $r = r_m$ is obtained by differentiating the equation of the parabola passing through the 3 points next to the walls. Thus, when the gas properties $C_p^{i,j}$ and $\lambda^{i,j}$, and the stream function $\psi^{i,j}$ are known, Equation (9) is reduced to a system of $n \times m$ linear algebraic equations with $n \times m$ temperature variables $T^{i,j}$. The convergence is achieved when the temperature variations are within a set error criterion. The details of the procedure are written below.

Momentum Equation

The finite difference form to Equation (7) is:

$$
\begin{aligned}
&\frac{\nu^{i,j-1} + \nu^{i,j}}{2K_r\, r^i\, (\Delta\theta)^2}\ \psi^{i,j-1} + \frac{\nu^{i,j} + \nu^{i,j+1}}{2K_r\, r^i\, (\Delta\theta)^2}\ \psi^{i,j+1} \\
&+ \frac{r^{i-1}\ \nu^{i-1,j} + r^i\ \nu^{i,j}}{2K_\theta\, (\Delta r)^2}\ \psi^{i-1,j} + \frac{r^i\ \nu^{i,j} + r^{i+1}\ \nu^{i,i+1}}{2K_\theta\, (\Delta r)^2}\ \psi^{i+1,j} \\
&- \left[\frac{\nu^{i,j-1} + 2\nu^{i,j} + \nu^{i,j-1}}{2K_r\, r^i\, (\Delta\theta)^2} + \frac{r^{i-1}\ \nu^{i-1,j} + r^i\ \nu^{i,j} + r^{i+1}\ \nu^{i+1,j}}{2K_\theta\, (\Delta r)^2} \right] \psi^{i,j} \\
&= g\ \frac{\cos\theta^j}{2\Delta\theta}\ (\rho^{i,j+1} - \rho^{i,j-1}) + \frac{r^i\ \sin\theta^j}{2\Delta r}\ (\rho^{i+1,j} - \rho^{i-1,j})
\end{aligned}
\tag{39}
$$

with $i = 2$ to $m-1$, and $j = 2$ to $n-1$.

The boundary condition equations are:

$$
\left.
\begin{aligned}
\psi^{1,j} &= 0 \\
\psi^{m,j} &= 0
\end{aligned}
\right\}\quad j = 1,n
$$

$$\psi^{i,1} = 0$$

$$i = 2, m-1$$

$$\psi^{i,n} = 0$$

Energy Equation

The finite difference form of Equation (9) is:

$$\left[\frac{(\psi^{i+1,j} - \psi^{i-1,j}) \; C_p^{i,j-1}}{4\,\Delta\theta\Delta r} - \frac{\lambda^{i,j} + \lambda^{i,j-1}}{2r^i\,(\Delta\theta)^2} \right] T^{i,j-1}$$

$$- \left[\frac{(\psi^{i+1,j} - \psi^{i-1,j}) \; C_p^{i,j+1}}{4\,\Delta\theta\Delta r} + \frac{K^{i,j+1} + K^{i,j}}{2r^i\,(\Delta\theta)^2} \right] T^{i,j+1}$$

$$- \left[\frac{(\psi^{i,j+1} - \psi^{i,j-1}) \; C_p^{i-1,j}}{4\,\Delta\theta\Delta r} + \frac{r^i\,\lambda^{i,j} + r^{i-1}\,\lambda^{i-1,j}}{2\,(\Delta r)^2} \right] T^{i-1,j}$$

$$+ \left[\frac{(\psi^{i,j+1} - \psi^{i,j-1}) \; C_p^{i+1,j}}{4\,\Delta\theta\Delta r} - \frac{r^{i+1}\,\lambda^{i+1,j} + r^i\,\lambda^{i,j}}{2\,(\Delta r)^2} \right] T^{i+1,j}$$

$$+ \left[\frac{r^{i+1}\,\lambda^{i+1,j} + 2r^i\,\lambda^{i,j} + r^{i-1}\,\lambda^{i-1,j}}{2\,(\Delta r)} + \frac{\lambda^{i,j+1} + 2\lambda^{i,j} + \lambda^{i,j+1}}{2r^i\,(\Delta\theta)^2} \right] T^{i,j} = 0 \qquad (42)$$

The boundary condition equations are:

$$T^{1,j} = T_1^j \quad \text{(given inner wall temperature)} \qquad (43\text{-a})$$

or

$$\left(h_1 + \frac{3\lambda^{1,j}}{2\Delta r}\right) T^{1,j} - \frac{2\lambda^{1,j}}{\Delta r} T^{2,j} + \frac{\lambda^{1,j}}{2\Delta r} T^{3,j} = h_1 T_a^j \quad \text{(convective boundary condition)} \qquad (43\text{-b})$$

$$T^{m,j} = T_2^j \quad \text{(given outer wall temperature)} \qquad (44\text{-a})$$

or

$$\left(h_2 + \frac{3\lambda^{m,j}}{2\Delta r}\right) T^{m,j} - \frac{2\lambda^{m,j}}{\Delta r} T^{m-1,j} + \frac{\lambda^{m,j}}{2\Delta r} T^{m-2,j} = h_2 T_o^j \quad \text{(convective boundary condition)} \qquad (44\text{-b})$$

with $j = 1$ *to* m,

$$T^{i,2} - T^{i,1} = 0$$

$$T^{i,n} - T^{i,n-1} = 0 \quad (45)$$

Solution of the set of linear algebraic equations

The solution starts with initial guesses of the temperatures. In the computer program, initial $T^{i,j}$'s are set equal to the mean temperature $\bar{T}$ of the insulation, which is a taken as the arithmetic average of the cold and hot wall temperatures. The gas properties $\nu^{i,j}$ and $\rho^{i,j}$ are evaluated at $\bar{T}$ and at the system pressure, using empirical equations to predict the properties. With numerical values of $\nu^{i,j}$ and $\rho^{i,j}$, Equation (39) is linearized.

To solve the set of n x m linear algebraic equations made up by Equations (39), (40) and (41), the coefficient matrix is formed in such a way as to obtain a band structure. The n x m grid points $z^{i,j}$ are reindexed as a one dimensional array z^k, with $z^1 = z^{1,1}$, $z^2 = z^{2,1}$, or $z^{(j-1)m+i} = z^{i,j}$. The variables $T^{i,j}$ are reindexed into T^K in a similar manner. The equations are numbered so that the first equation is written at the point z^1, second at z^2, etc. In this way, the coefficient matrix has a band sctructure, with m upper and m lower codioganals, and it is stored in a one dimensional array of size $s = m \times n\,(2m+1) - \frac{m(m+1)}{2}$. The size of the general coefficient matrix for the same system would be $(nxm)^2$. The band structure thus allows the solution of the large sets of linear equations necessary to obtain accurate numerical results.

For the solution of the equations, subroutine GELB from IBM SSP (scienticif Subroutine Package) is used. In GELB, solution is done by me s of Gauss elimination with column pivoting only, in order to preserve band structure in remaining coefficient matrices.

The structure of the coefficient matrix of the momentum equation and boundary conditions for a grid of 4 x 4 nodes is shown in Figure 5. Energy equation has a similar matrix structure.

The first solution of the momentum equation, with $T^{i,j} = \bar{T}$, results in $\psi^{i,j} = 0$ since $\frac{\partial \rho^{i,j}}{\partial r} = \frac{\partial \rho^{i,j}}{\partial \theta} = 0$. Next, the energy equation is solved, and with $\psi^{i,j} = 0$ it yields the solution of the conduction problem, with the gas properties evaluated at $\bar{T}$. The properties $\nu^{i,j}$ and $\rho^{i,j}$ for the next solution of the momentum equation are evaluated at $T^{i,j}$'s obtained by a weighted average of the last two set of temperatures. For the next solution of the energy equation, a weighted average of the last two $\psi^{i,j}$'s and gas properties evaluated at $T^{i,j}$ are used. The procedure is continued until convergence of temperatures is obtained. For a grid at 20 x 20, at Ra numbers smaller than 1000, the temperatures converged within 0.1°C after about 15 itirations. At $3000 > Ra > 1000$, the temperatures fluctuated around the solution by about 1°C. Approximatly at Ra numbers greater than 3000, convergence difficulties were encountered. The convergence should improve at finer grid sizes.

The solution of a problem with 20 x 20 nodes required a storage at 200 K (about 130 K to store the coefficient matrix in double precision), and 5 minutes of CPU time for 15 itirations on the I.E.A.'s IBM 370/55.

The flow chart of the computer progrm FINS is shown in Figure 6.

Equations

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	X															
2		X														
3			X													
4				X												
5					X											
6		X			X	X	X			X						
7			X			X	X	X				X				
8								X								
9									X							
10						X			X	X	X			X		
11										X	X	X				
12												X				
13													X			
14														X		
15															X	
16																X

Figure 5 – Coefficient Matrix for a grid of 4 x 4 nodes for the Momentum Equation and the Boundary Conditions. x's are non-zero elements.

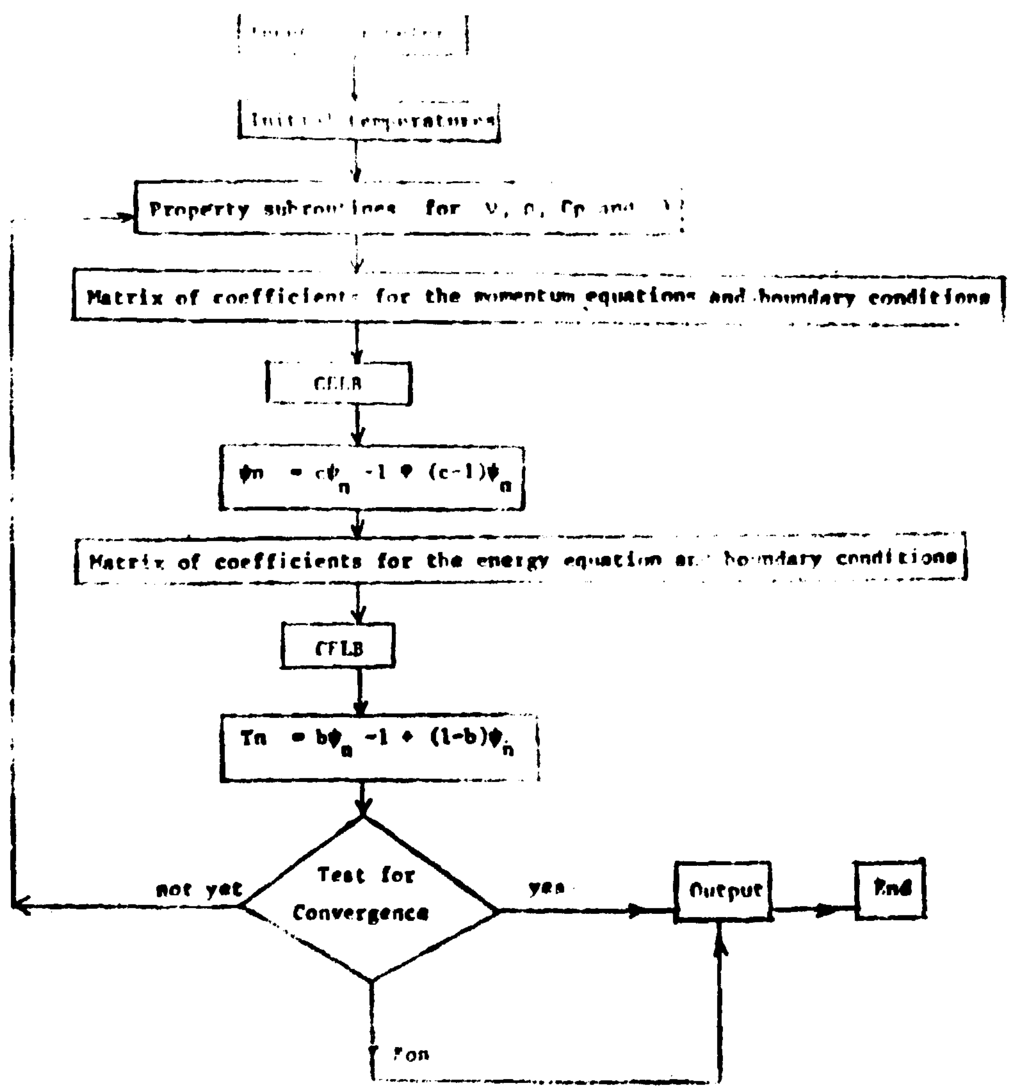


Figure 8 Flow Chart for FINS

For the weight factors a and b several combinations have been tried. For small Ra numbers, a = b = 0 gives rapid convergence. At high Ra numbers, a ≥ .5, b ≥ 0 is necessary. The final version of FINS is with a = .5, and b = .0.

3.2 – Vertical Cylinder

The finite difference equations for the vertical cylinder are derived similarly to horizontal cylinder (Figure 7).

Momentum Equation (Equation (16))

$$\frac{\nu^{i,j} + \nu^{i,j-1}}{2K_r\, r^i\, (\Delta x)^2}\, \psi^{i,j-1} + \frac{\nu^{i,j+1} + \nu^{i,j}}{2K_r\, r^i\, (\Delta x)^2}\, \psi^{i,j+1}$$

$$+ \frac{\dfrac{\nu^{i,j}}{r^i} + \dfrac{\nu^{i-1,j}}{r^{i-1}}}{2K_x\, (\Delta r)^2}\, \psi^{i-1,j} + \frac{\dfrac{\nu^{i+1,j}}{r^{i+1}} + \dfrac{\nu^{i,j}}{r^i}}{2K_x\, (\Delta r)^2}\, \psi^{i+1,j}$$

$$- \left[\frac{\nu^{i,j+1} + 2\,\nu^{i,j} + \nu^{i,j-1}}{2K_r\, r^i\, (\Delta x)_2} + \frac{\dfrac{\nu^{i+1,j}}{r^{i+1}} + 2\,\dfrac{\nu^{i,j}}{r^i} + \dfrac{\nu^{i-1,j}}{r^{i-1}}}{2K_x\, (\Delta r)^2} \right] \psi^{i,j}$$

$$= \frac{g}{2\,\Delta r}\left(\rho^{i+1,j} - \rho^{i-1,j}\right) \qquad (46)$$

with $i = 2$ to $m-1$ and $j = 2$ to $n-2$.

Boundary Conditions

Equations corresponding to $\psi = 0$ at $x = 0$ and $x = L$ are:

$$\psi^{i,1} = 0 \qquad (47)$$

$$\psi^{i,n} = 0$$

with $i = 1$ to m, and at $r = r_1$

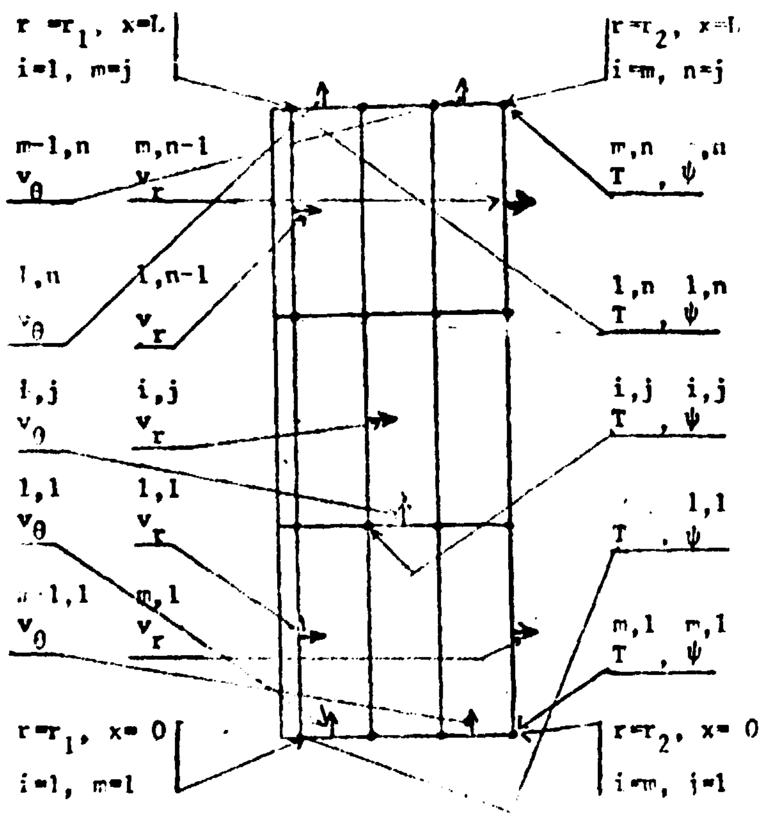


Figure 7 – Vertical Cylinder, Finite Difference Grid

$$\psi^{1,j} = 0 \quad \text{(solid wall)} \tag{48-a}$$

or

$$-\left(\frac{1}{\Delta r} + \frac{2aK}{(\Delta x)^2 K_c}\right)\psi^{1,j} + \frac{1}{\Delta r}\psi^{2,j} + \frac{aK}{(\Delta x)^2 K_c}\psi^{1,j-1} + \frac{aK}{(\Delta x)^2 K_c}\psi^{1,j+1}$$

$$= \frac{rK^1}{\mu^{1,j}}\left(g\rho^{1,j} + \frac{\partial p_0}{\partial x}\right) \tag{48-b}$$

with $j = 2, n-1$, and at $r = r_2$

$$\psi^{m,j} = 0 \tag{49}$$

with $i = 1, m$

Equation (48b) corresponds to Equation (19) which is used in the case permeation flow.

Energy Equation (Equation (18))

$$\left[\frac{(\psi^{i+1,j} - \psi^{i-1,j})\, C_p^{i,j-1}}{4\,\Delta x\Delta r} - \frac{r^i\,(\lambda^{i,j} + \lambda^{i-1,j})}{2\,(\Delta x)^2}\right] T^{i,j-1}$$

$$-\left[\frac{(\psi^{i+1,j} - \psi^{i-1,j})\, C_p^{i,j+1}}{4\,\Delta x\Delta r} - \frac{r^{i+1}\,\lambda^{i+1,j} + r^i\,\lambda^{i,j}}{2\,(\Delta r)^2}\right] T^{i,j+1}$$

$$-\left[\frac{(\psi^{i,j+1} - \psi^{i,j-1})\, C_p^{i-1,j}}{4\,\Delta x\Delta r} + \frac{r^i\,\lambda^{i,j} + r^{i-1}\,\lambda^{i-1,j}}{2\,(\Delta r)^2}\right] T^{i-1,j}$$

$$+\left[\frac{(\psi^{i,j+1} - \psi^{i,j-1})\, C_p^{i+1,j}}{4\,\Delta x\Delta r} - \frac{r^{i+1}\,\lambda^{i+1,j} + r^i\,\lambda^{i,j}}{2\,(\Delta r)^2}\right] T^{i+1,j}$$

$$+\left[\frac{r^{i+1}\,\lambda^{i+1,j} + 2r^i\,\lambda^{i,j} + r^{i-1}\,\lambda^{i-1,j}}{2\,(\Delta x)^2} + \frac{r^i\,(\lambda^{i,j+1} + 2\lambda^{i,j} + \lambda^{i,j-1})}{2\,(\Delta x)^2}\right] T^{i,j} = 0 \tag{50}$$

With $i = 2$ to $m,1$ and $j = 2$ to $n-1$

Boundary Conditions

$$T^{1,j} = T_1^j \quad \text{(given hot wall temperature)}$$

$$T^{m,j} = T_2^j \quad \text{(given cold wall temperature)} \qquad (51\text{-a})$$

or

$$\left(h_1 + \frac{3\lambda^{1,j}}{2\,\Delta r}\right) T^{1,j} - \frac{2\lambda^{1,j}}{\Delta r} T^{2,j} + \frac{\lambda^{1,j}}{2\,\Delta r} T^{3,j} = h_1\, T_g$$

$$\left(h_2 + \frac{3\lambda^{m,j}}{2\,\Delta r}\right) T^{m,j} - \frac{2\lambda^{m,j}}{\Delta r} T^{m-1,j} + \frac{\lambda^{m,j}}{2\,\Delta r} T^{m-2,j} = h_2\, T_o \quad \text{(convective boundary conditions)} \qquad (51\text{-b})$$

with $j = 1$ to n, and to express $\frac{\partial T}{\partial x} = 0$ at $x = 0$ and $x = L$:

$$T^{i,2} - T^{i,1} = 0$$

$$T^{i,n} - T^{i,n-1} = 0 \qquad (52)$$

with $i = 2,\ m-1$

The solution of the finite difference equations is similar to that for the horizontal cylinder.

3.3 – Rectangular Geometries

Momentum Equation

Finite difference form of Equation (31) is (Figure 8):

$$\frac{\nu^{i,j} + \nu^{i,j-1}}{2K_y\,(\Delta x)^2}\,\psi^{i,j-1} + \frac{\nu^{i,j+1} + \nu^{i,j}}{2K_r\,(\Delta x)^2}\,\psi^{i,j+1}$$

$$+ \frac{\nu^{i,j} + \nu^{i-1,j}}{2K_x\,(\Delta y)^2}\,\psi^{i-1,j} + \frac{\nu^{i+1,j} + \nu^{i,j}}{2K_x\,(\Delta y)^2}\,\psi^{i+1,j}$$

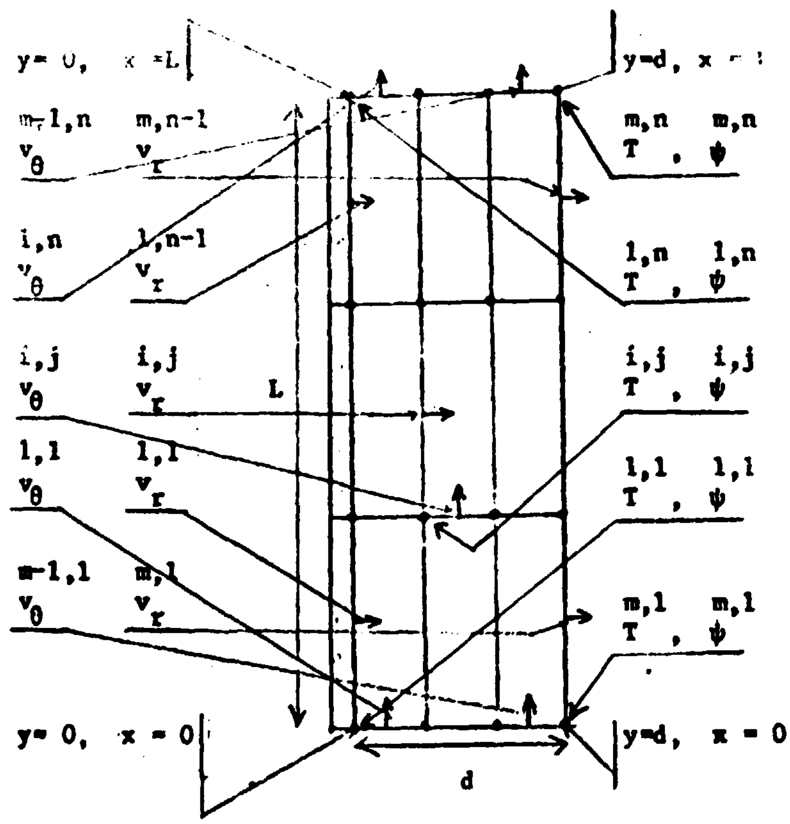


Figure 8 – Rectangular Geometry, Finite Difference Grid

$$- \left[\frac{\nu^{i,j+1} + 2\nu^{i,j} + \nu^{i,j-1}}{2K_y (\Delta x)^2} + \frac{\nu^{i+1,j} + 2\nu^{i,j} + \nu^{i-1,j}}{2K_x (\Delta y)^2} \right] \psi^{i,j} \qquad (53)$$

Boundary Conditions

Equation corresponding to $\psi = 0$ at $x = 0$ and $x = L$ are:

$$\psi^{i,1} = 0$$

$$\psi^{i,n} = 0 \qquad (54)$$

with $i = 1$ to m, and $r = r_1$

$$\psi^{1,j} = 0 \quad \text{(solid wall)} \qquad (55\text{-a})$$

or

$$- \left(\frac{1}{\Delta y} + \frac{2\,aK}{K_c (\Delta x)^2} \right) \psi^{1,j} + \frac{1}{\Delta y} \psi^{2,j} + \frac{aK}{K_c (\Delta x)^2} \psi^{1,j-1} + \frac{aK}{K_c (\Delta x)^2} \psi^{1,j+1}$$

$$= \frac{K}{\nu^{1,j}} \left(- \rho^{1,j} g_x + \frac{\partial p_o}{\partial x} + a g_y \right) \qquad (55\text{-b})$$

with $j = 2$ to $n-1$, and at $r = r_2$

$$\psi^{m,j} = 0 \qquad (56)$$

with $i = 1$ to m.

Equation (55-b) corresponds to Equation (34) which is used in the case of permeation flow through the cover plate.

Energy Equation

Finite difference form of Equation (33) is:

$$\left[\frac{(\psi^{i+1,j} - \psi^{i-1,j})\, C_p^{i,j-1}}{4\,\Delta x \Delta y} - \frac{\lambda^{i,j} + \lambda^{i-1,j}}{2\,(\Delta x)^2} \right] T^{i,j-1}$$

$$- [\frac{(\psi^{i+1,j} - \psi^{i-1,j}) C_p^{i,j+1}}{4 \Delta x \Delta y} + \frac{\lambda^{i,j+1} + \lambda^{i,j}}{2 (\Delta x)^2}] T^{i,j+1}$$

$$- [\frac{(\psi^{i,j+1} - \psi^{i,j-1}) C_p^{i-1,j}}{4 \Delta x \Delta y} + \frac{\lambda^{i,j} + \lambda^{i-1,j}}{2 (\Delta y)^2}] T^{i-1,j}$$

$$+ [\frac{(\psi^{i,j+1} - \psi^{i,j-1}) C_p^{i+1,j}}{4 \Delta x \Delta y} - \frac{\lambda^{i+1,j} + \lambda^{i,j}}{2 (\Delta y)^2}] T^{i+1,j}$$

$$+ [\frac{\lambda^{i+1,j} + 2\lambda^{i,j} + \lambda^{i-1,j}}{2 (\Delta y)^2} + \frac{\lambda^{i,j+1} + 2\lambda^{i,j} + \lambda^{i,j-1}}{2 (\Delta x)^2}] T^{i,j} = 0 \qquad (57)$$

Boundary Conditions

$$T^{1,j} = T_1^j \quad \text{(given hot wall temperature)}$$

(58-a)

$$T^{m,j} = T_2^j \quad \text{(given cold wall temperature)}$$

or

$$(h_1 + \frac{3\lambda^{1,j}}{2 \Delta y}) T^{1,j} - \frac{2\lambda^{1,j}}{\Delta y} T^{2,j} + \frac{\lambda^{1,j}}{2 \Delta y} T^{3,j} = h_1 T_e$$

(58-b)

$$(h_2 + \frac{3\lambda^{m,j}}{2 \Delta y}) T^{m,j} - \frac{2\lambda^{m,j}}{\Delta y} T^{m-1,j} + \frac{\lambda^{m,j}}{2 \Delta y} T^{m-2,j} = h_2 T_e \quad \text{(convective boundary condition)}$$

with $j = 1$ to n, and to express $\frac{\partial T}{\partial x} = 0$ at $x = 0$ and $x = L$:

$$T^{i,2} - T^{i,1} = 0$$

(59)

$$T^{i,n} - T^{i,n-1} = 0$$

with $i = 2$ to $m - 1$.

The solution of the finite difference equations is similar to that of the horizontal cylinder.

4 – RESULTS

The program FINS has been run for the cylindrical and rectangular geometries with the various boundary conditions discussed earlier. The fluid has been taken as carbon dioxide, with pressures from 1 to 100 bar and mean temperatures from 350°K to 550°K an effective thermal conductivity has been calculated from the velocity and temperature distribution results. The effective Nusselt number has been correlated with the Rayleigh and Peclet number in the case of a permeable hot wall, and with Rayleigh number only in the case of a solid hot wall. The results for the rectangular geometries showed close agreement with a previously published correlation[6].

4.1 – Horizontal Cylinder

The parameters covered the following range:

Wall temperature T_2:	316 – 400° K
Hot wall temperature T_1:	365 – 700° K
Temperature difference $T_1 - T_2$:	50 – 300° K
Pressure:	13 – 100 bar
Permeability $(K_\theta = K_r)$:	1.E-8 – 1.E-11 m^2
Heat transfer coefficient h_1:	5 – 1000 $\frac{w}{m^2 \cdot {}^\circ K}$
Heat transfer coefficient h_2:	5 – 1000 $\frac{w}{m^2\ K}$
Inside radius r_1:	.05 – .09 m
Outside radius r_2:	.075 – .01 m
Insulation thickness $r_2 - r_1$:	.01 – .05
Aspect ratio A:	4.71 – 30.0

Velocity and Temperature Distribution

At values of parameter M $(M = \frac{Ra}{A})$ less than 1, the computations showed that the gas does not circulate. Therefore, the isotherms are concentric cylinders, corresponding to the case of heat transfer by conduction only. At $M > 1$, the gas flows upwards (Figure 9-a) along the hot wall and downward along the cold wall. The corresponding temperature distribution deviates considerably from the conduction problem (Figure 9-b). The mass velocity components m_r and m_θ are shown on Figures 9-c and 9-d.

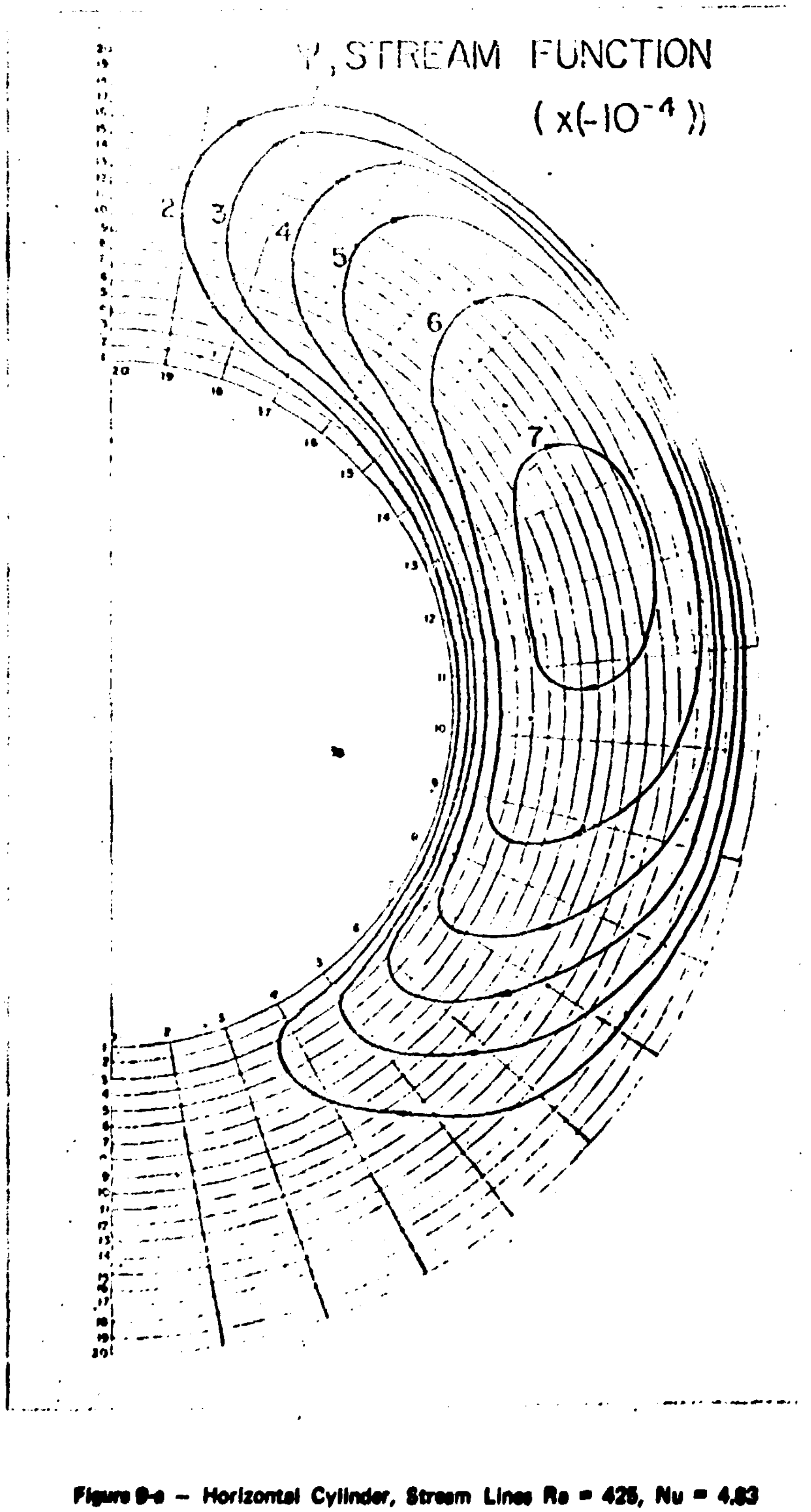


Figure 9-e – Horizontal Cylinder, Stream Lines Ra = 425, Nu = 4.83

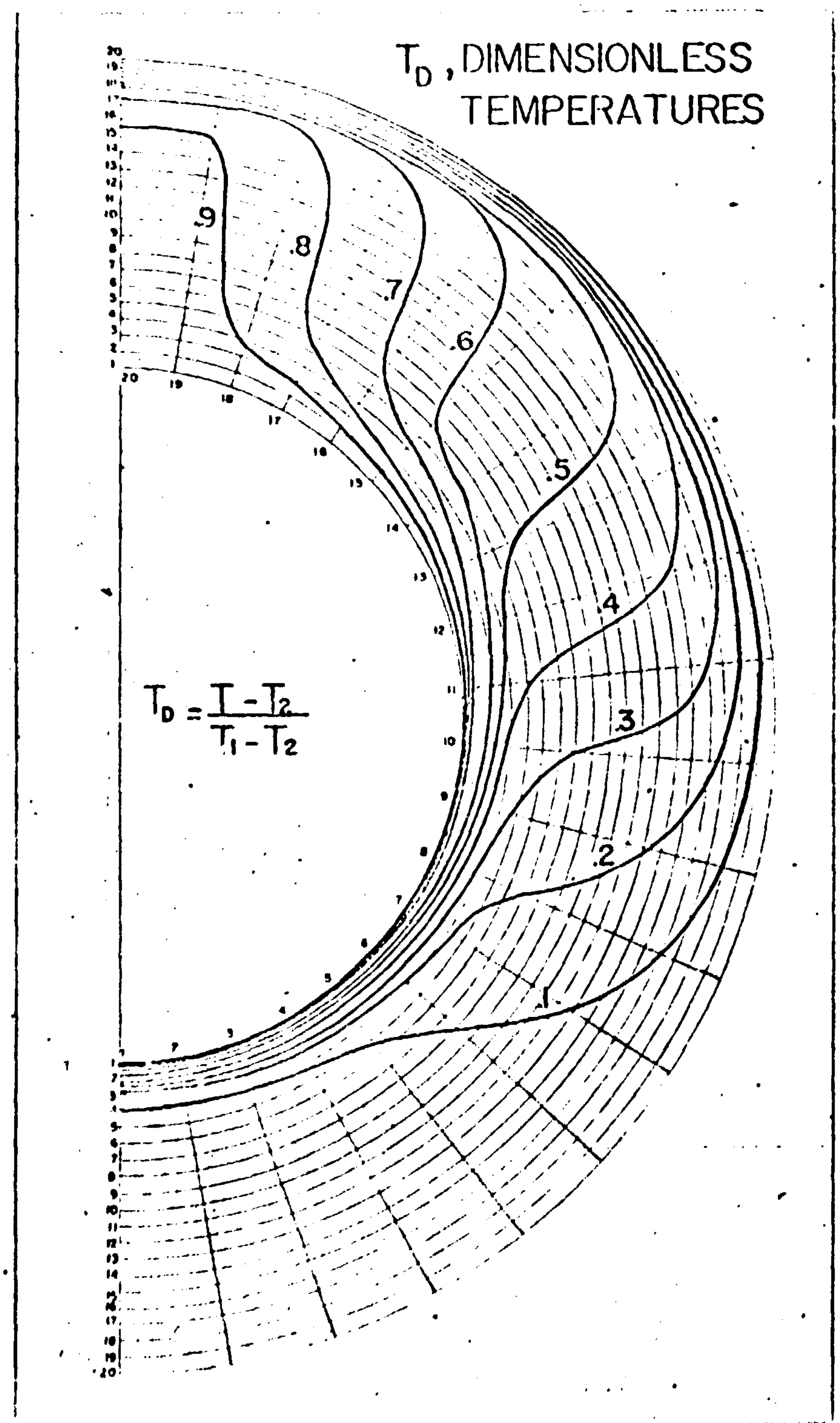
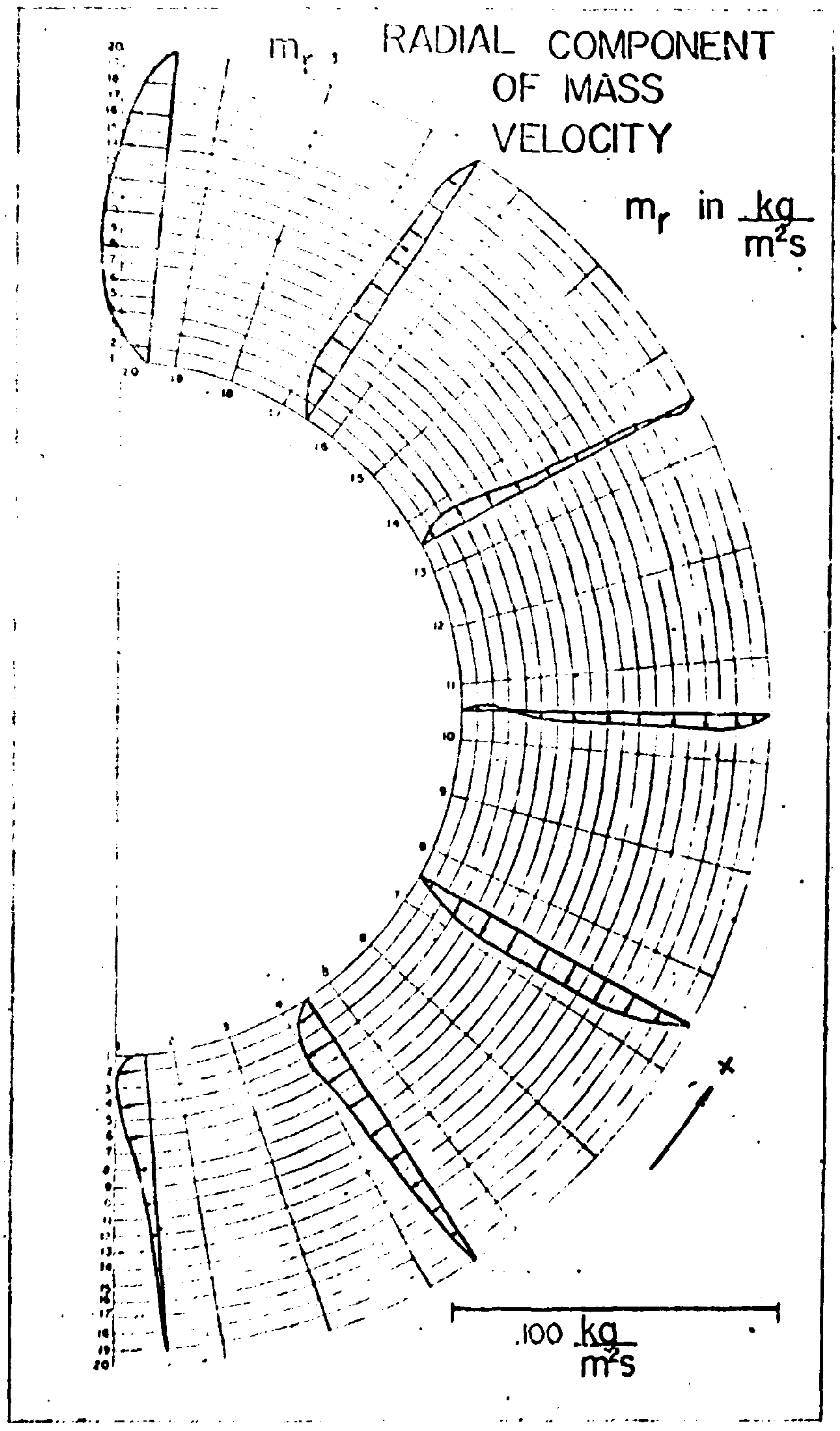


Figure 9 b - Horizontal Cylinder, Dimensionless Temperatures Ra = 425 Nu = 4.83



—Figure 9-c – Horizontal Cylinder, Radial Component of Mass Velocity, Ra = 425, Nu = 4.83

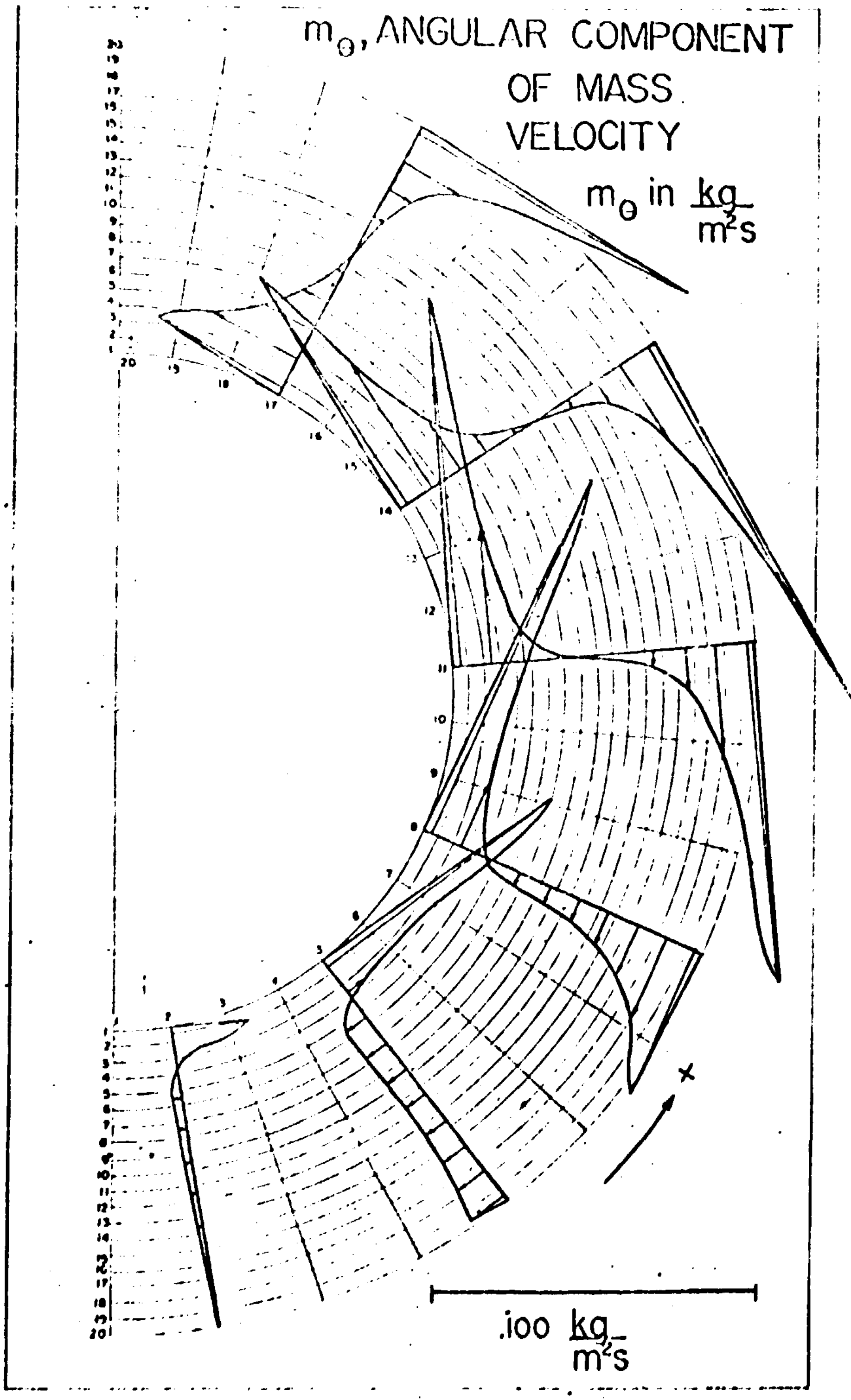


Figure 9-d – Horizontal Cylinder, Angular Component of Mass Velocity, Ra = 425, Nu = 4.83

Effective Thermal Conductivity

The effective thermal conductivity λ^e is defined by

$$\lambda^e = \frac{Q \ln (r_2/r_1)}{(T_1 - T_2)\,\Pi} \tag{60}$$

where Q is the heat flow through the insulation. Q is calculated at each radial position r^i from

$$Q_i = \sum_{j=1}^{J} r^i \,\Delta\theta \,(\rho^{i,j} v_r^{i,j} - \lambda^{i,j} \frac{\partial T^{i,j}}{\partial r}) \tag{61}$$

where i and j are the node indices in the radial and angular directions respectively. At the walls (i = 1 and i = n) the temperature gradient $\frac{\partial T}{\partial r}$ is evaluated from a second degree approximation of the temperature profile near the wall. The convective terms v_r are zero in this case. At other values of r, $\frac{\partial T}{\partial r}$ is approximated by central differences. The computed Q_i's differed by about ± 5%, and the heat flow has been taken as the arithmetic average of the hot and cold wall values. When $\frac{\partial T}{\partial r}$ was evaluated with a linear approximation of the temperature profile near the walls, lower heat flows (up to 30%) were obtained. This is explained by steep temperature gradients at the walls which are not well approximated by a linear temperature profile.

The overall Nusselt number Nu is defined as

$$Nu = \frac{\lambda^e}{\lambda}$$

where λ is the thermal [illegible] insulation (gas + fibre).

The overall Nusselt number correlated with an equation of the form $Nu = c\,(\frac{Ra}{A})^n$. The result was (Figure 10)

$$Nu = .48 \,(\frac{Ra}{A})^{.5} \qquad Ra > 1$$

$$Nu = 1 \qquad Ra < 1 \tag{62}$$

Where the Rayleigh number Ra is defined by

$$Ra = \frac{g(T_H - T_C)\, d\;\; K\, \beta}{\nu_f\, \alpha_f}$$

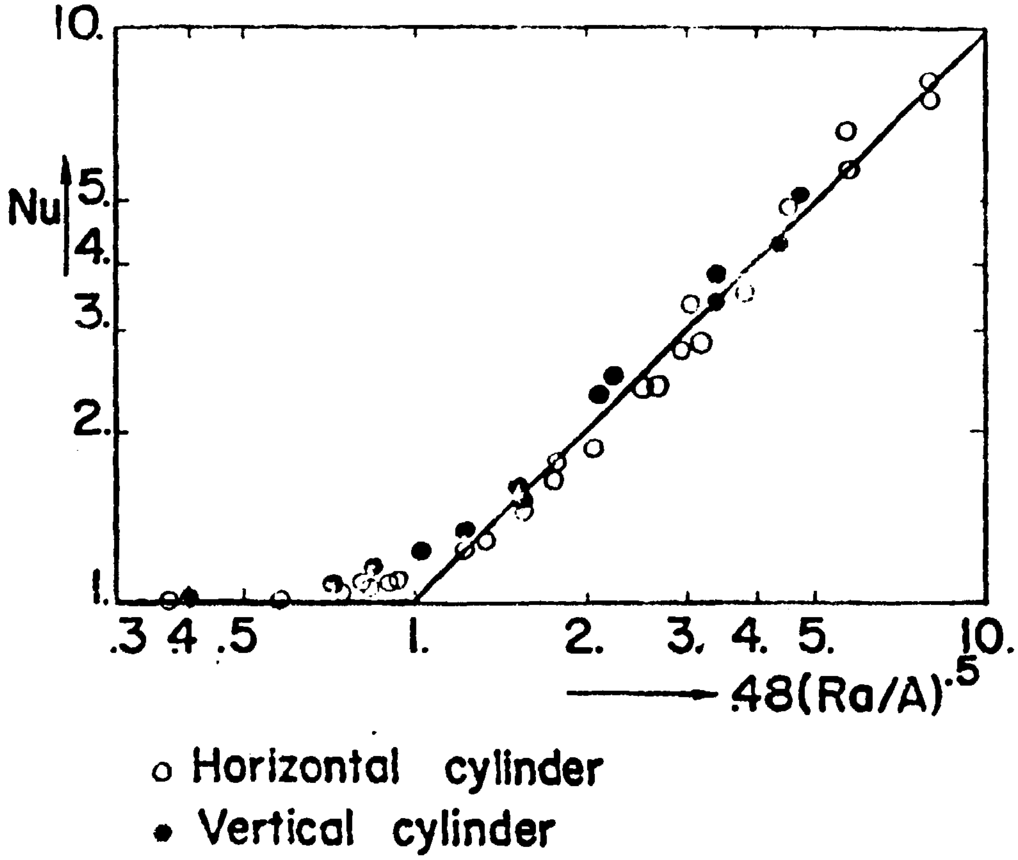


Figure 10 – Correlation of Overall Nusselt Number for Horizontal Cylinder and Vertical Cylinder Without Permeation Flow.

Where $K = K_r$ for cylindrical and $K = K_y$ for rectangular geometries.

The computed values of Nu showed a spread of ± 10% around the curve $Nu = .48(\frac{Ra}{A})^{.5}$. The lower values are from runs with 13 and 25 bars and higher ones from runs at 80 and 100 bars.

The variation of the local Nusselt number Nu_j with θ at the cold wall is shown in Figure 11.

$\frac{Nu_j}{Nu}$ is obtained from

$$\frac{Nu_j}{Nu} = \frac{\lambda^{m,j} \overline{\frac{\partial T^{m,j}}{\partial r}}}{Q/(n-1)} \tag{63}$$

4.2 – Vertical Cylinder

The range of input parameters was similar to that for the horizontal cylinder. In addition the fallo~~ing, permeation flow parameters have been used:

Cover plate permeability K_c.	1.E-8 – 1.E-12 m^2
Pressure gradient in the gas stream $\frac{\partial p_o}{\partial x}$:	$-1{,}5$ to $+1.5$ $\frac{bar}{m}$
Cover plate thickness:	.002 m
Prandtl Number:	.40 – .90

Closed Hot Wall

The computed velocity and temperature distributions (Figures 12-a and 12-b) found to be similar to those discussed earlier for the horizontal cylinder. λ^* was calculated from

$$\lambda^* = \frac{Q \ln (r_2/r_1)}{2(T_1 - T_2)\, L\, \Pi} \tag{64}$$

Where L is the hight of the cylinder. 1, the heat flux through the isolation has been again computed at each radial position r^i from

$$Q_i = \sum_{j=1}^{n-1} 2\Pi \Delta x r^i \left(\rho \; v_r^{i,j} - \lambda^{i,j} \frac{\partial T^{i,j}}{\partial r}\right) \tag{65}$$

Where i and j are the node indices in the radial and vertical directions respectively. $\frac{\partial T}{\partial r}$ evaluation is similar to that for the horizontal cylinder.

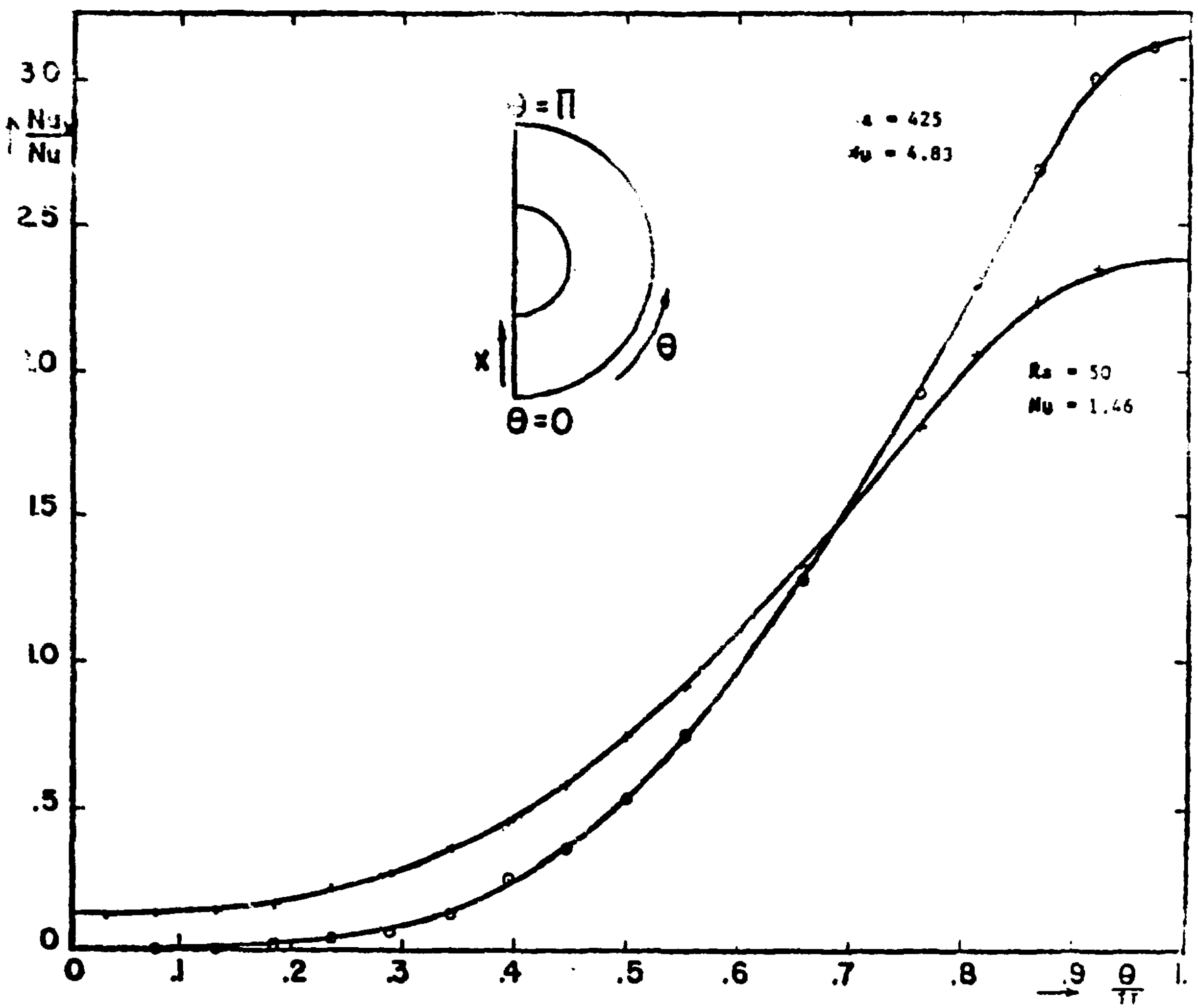


Figure 11 — Horizontal Cylinder, Variation of the Local Nusselt Number Along the Cold Wall

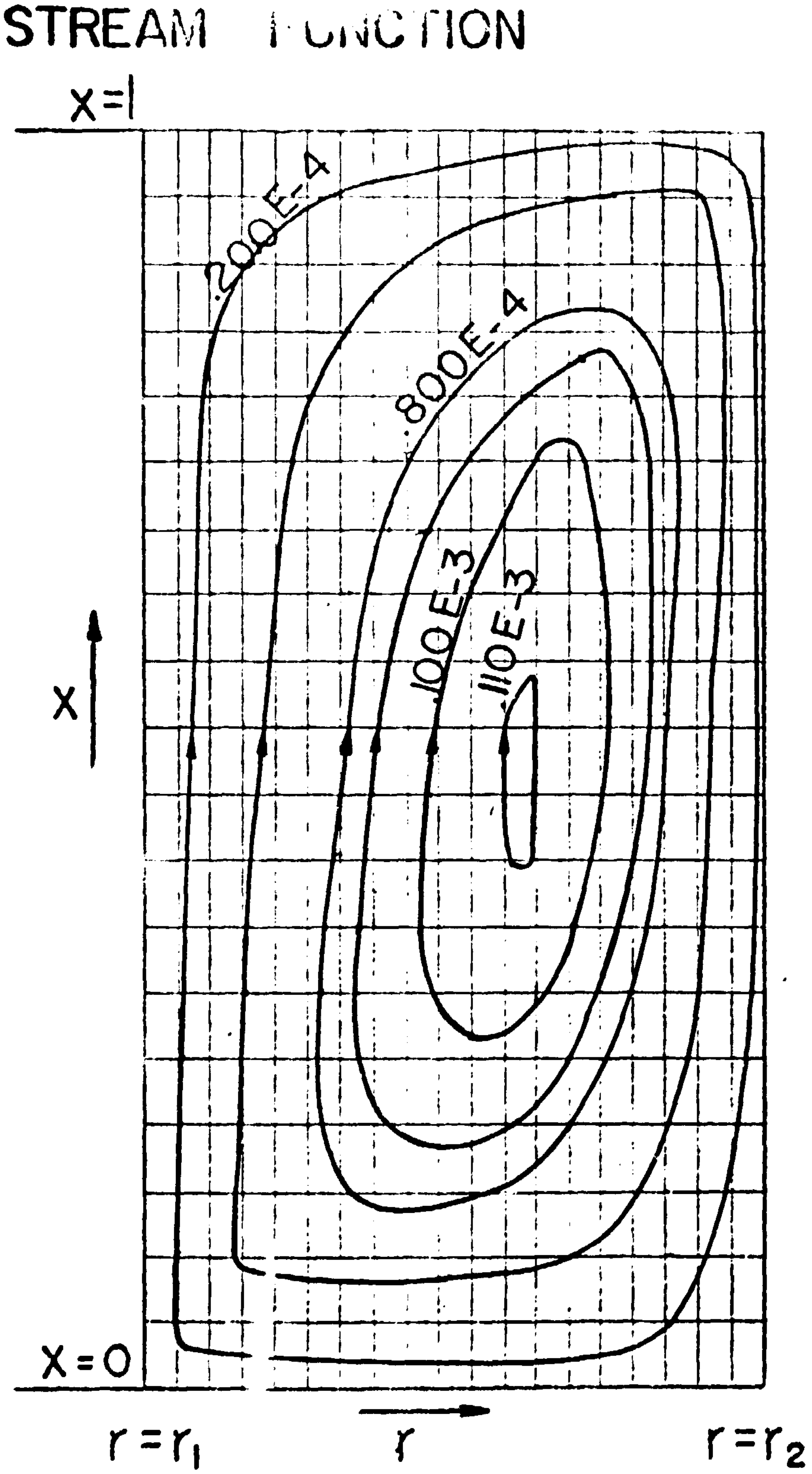


Figure 12-e – Vertical Cylinder, Str am Lines Ra = 204, Nu = 2.32. The scale in the radial direction is 5 times the scale in the vertical direction.

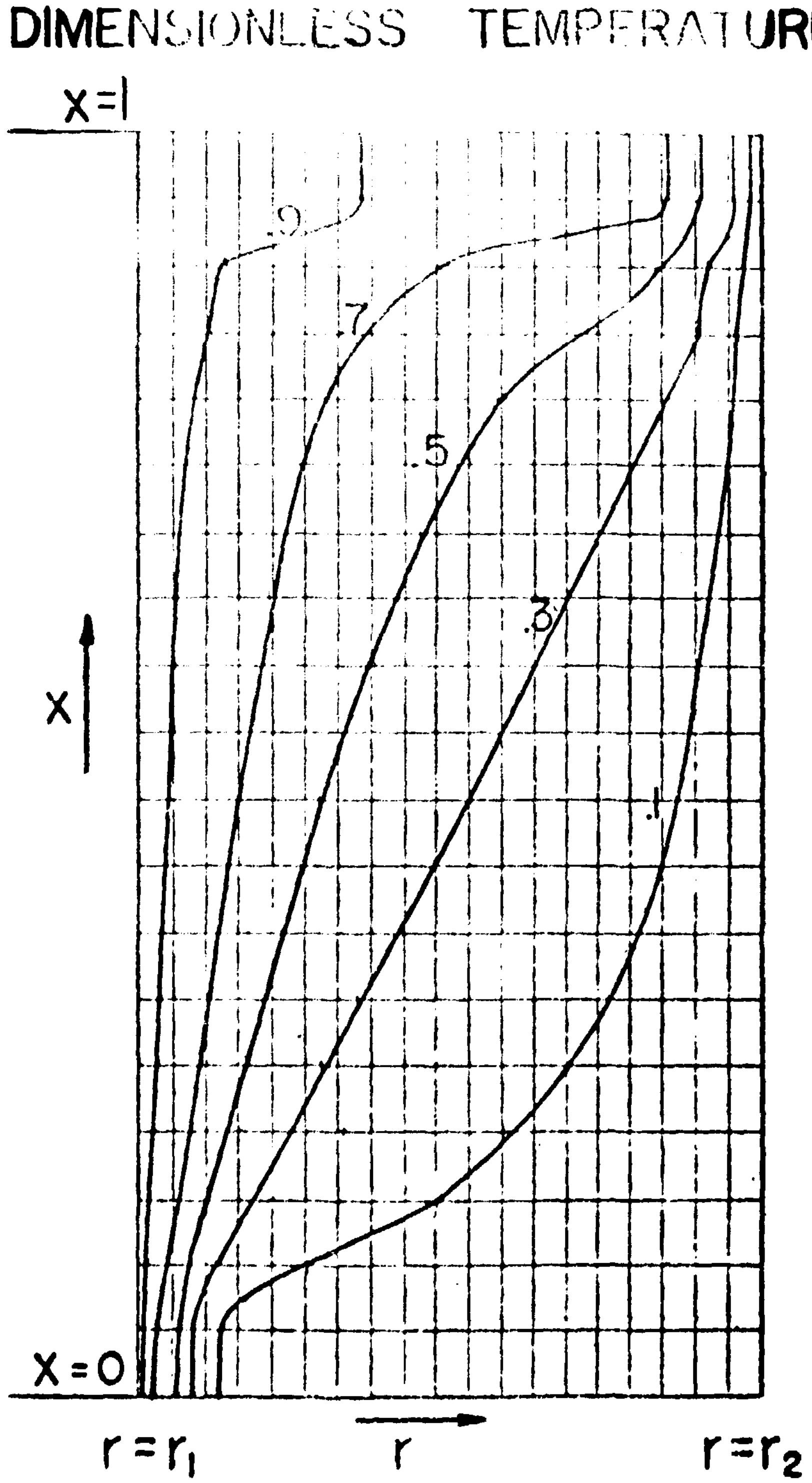


Figure 12-b – Vertical Cylinder, Dimensionless Temperatures Ra = 204, Nu = 2.32. The scale in the radial direction is 5 times the scale in the vertical direction.

The correlation of the overall Nusselt number with the parmeter $\frac{Ra}{A}$ (Figure 10) results in the same equation (again within ± 10%) as for the horizontal cylinder which is

$$Nu = .48 \left(\frac{Ra}{A}\right)^{.5} \tag{66}$$

Figure 13 shows the variation of the cold face local Nusselt number Nu_j in the vertical direction. The ratio Nu_j/Nu was again obtained from Equation (63).

Permeable Hot Wall

When the hot wall is permeable, the velocity and temperature distribution in the insulation depends on the Rayleigh and Reynolds numbers. The Reynolds number is taken as positive when the hot gas in the inner pipe flows downward and as negative when it flows upward. The Reynolds number is defined by

$$Re = \frac{K_x \, (\partial p_o/\partial x - \rho_o g_x)\, d}{\mu_r \, \nu_o} \tag{67}$$

Where the subscript o refers to free stream and r to average insulation temperatures. The pressure gradient $\partial p_o/\partial x$ can be evaluated from friction factor correlations for flow inside pipes.

Figures 14-a and 14-b show the stream lines and dimensionless temperatures for Ra = 51 and Re = 0. The Nusselt number is 2.09. The outside gas enters the insulation layer in the upper part, flows downward along the cold wall leaves in the lower part, fallowing a circultion pattern close to that observed in the case of a closed hot wall. Comparison of Figures 12-b and 14-b shows that temperature distributions are also similar. When Ra = 5.1 and Re = 161, flow and temperature fields characteristic of forced convection are seen (Figures 15-a and 15-b). They are different from what is encountered for natural circulation, but the flow is still downward along the cold wall. Figures 16-a and 16-b show a case where natural and forced convection (Ra = 51 and Re = 29) patterns are both present.

When Re is negative (upward gas flow in the pipe), forced and natural convection act in opposite directions. For Ra = 5.1 and Re = − 132 (Figures 17-a and 17-b), the gas enters the insulation in the lower part, flows upward along the cold wall, and leaves in the upper part. The difference in the temperature profiles corresponding to positive and negative Reynolds numbers (with negligible natural circulation effects) can be observed comparing Figures 17-b and 15-b. When Ra = 51 and Re = 86. Figure 18-a shows a forced convection pattern on the hot wall side and a natural circulation pattern on the cold wall side, acting in opposite direction; which results in the almost parallel and equidistant isothermes, which Nu = 1.17 (Figure 18-b). For Ra = 126 and Re = -128 (Figures 19-a and 19-b), the beginning of forced convection permeation flows can be observed near the hot wall, while the rest of the flow field is dominated by natural circulation. Finally, for Re = 51 and Re = -28, the forced convection is not observable any more (Figures 20-a and 20-b), but it reduces the amount of natural convection which results in a Nu of 1.58 (Nu = 2.09 for Ra = 50 and Ra = 0).

The variation of local Nusselt number on the cold wall is shown on Figure 21. For Re > 0, Nu is maximum on top and decreases towards the bottom. For Re < 0, and Ra << Re, Nu is maximum on the bottom and decreases towards the top. For Re negative and Ra of comparable magnitude, variation of the local Nusselt number show a mixed forced-natural convection pattern.

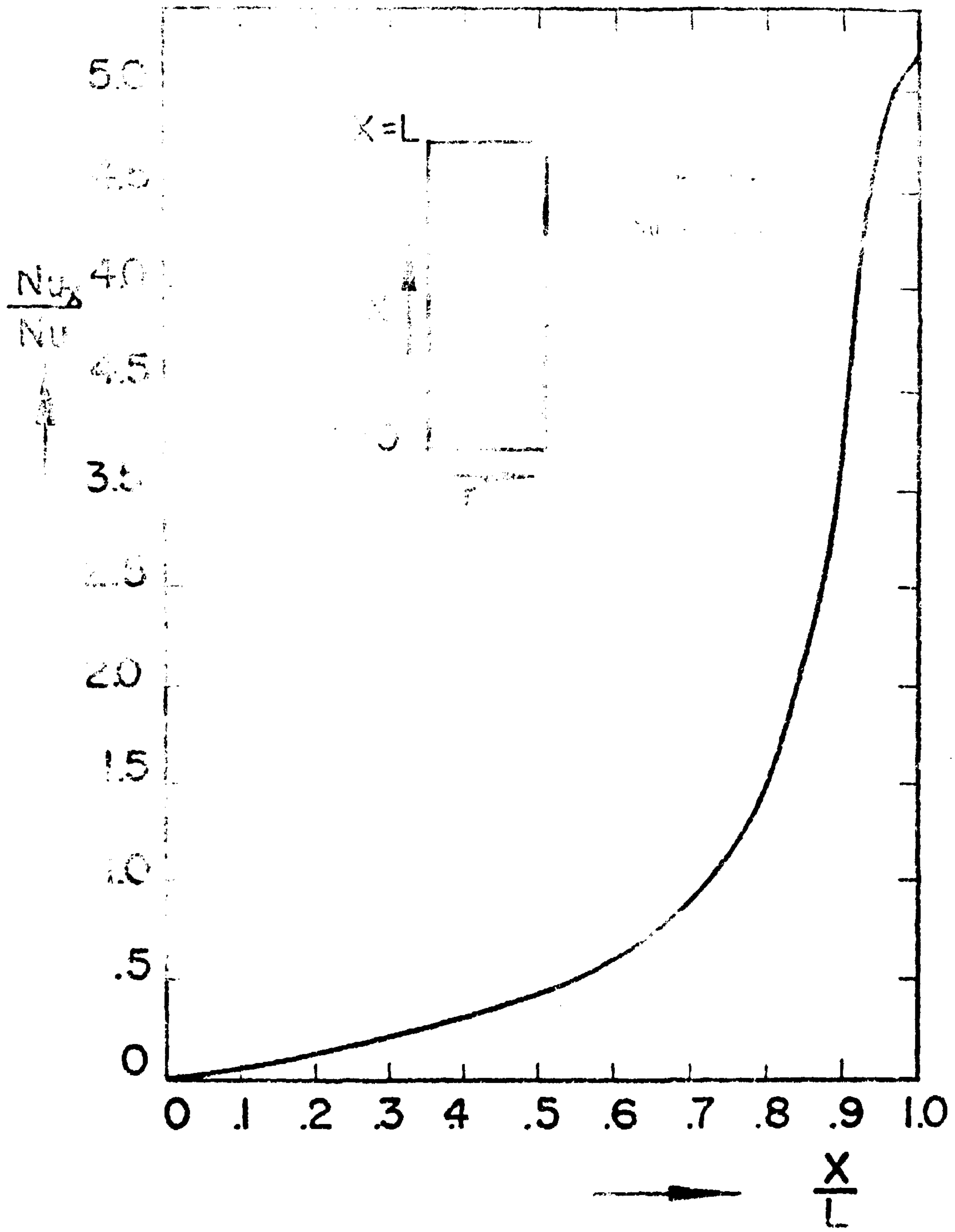


Figure 13 – Vertical Cylinder Without Permeation Flow, Variation of the Local Nusselt Number Along the Cold Wall

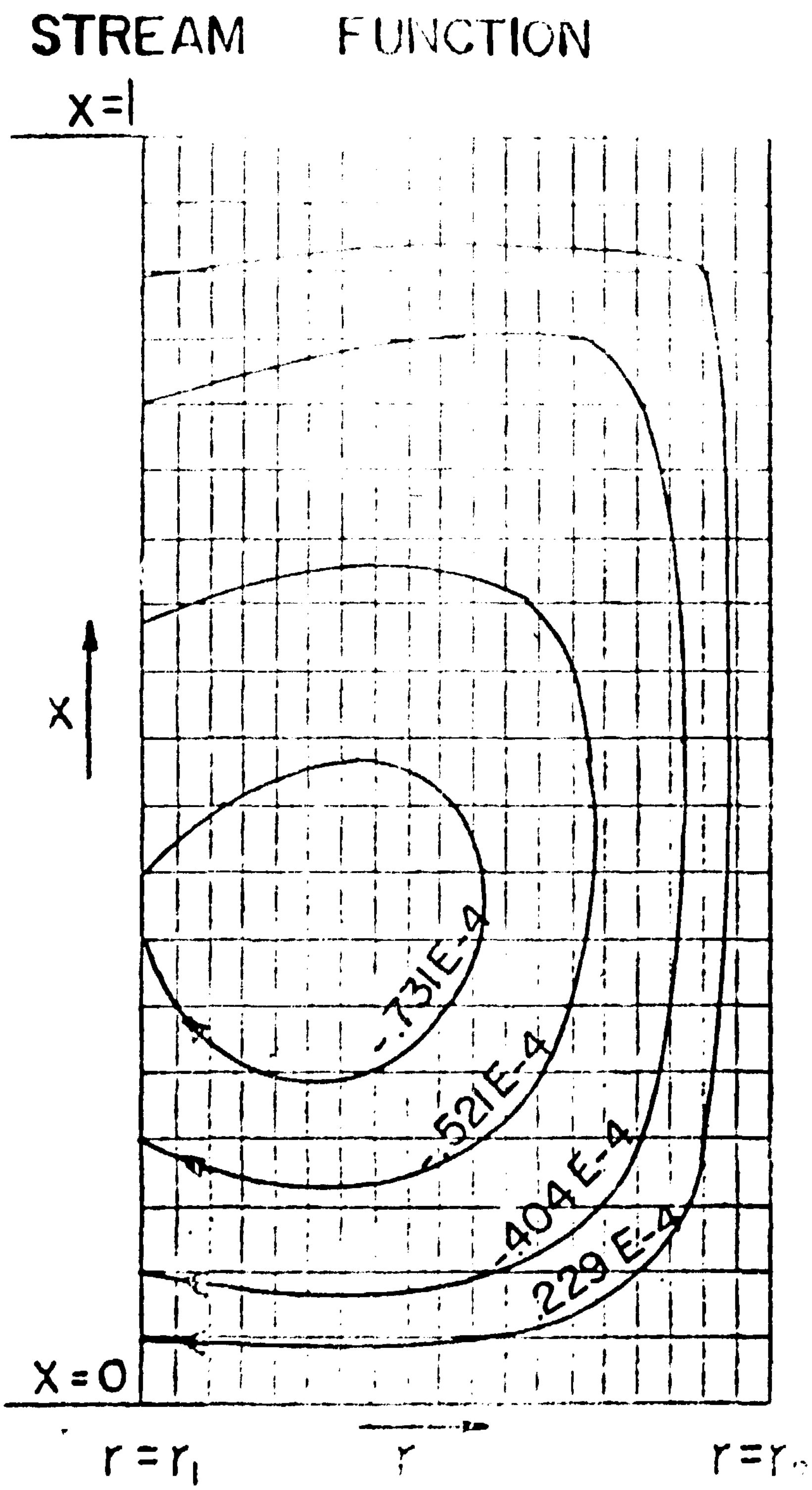


Figure 14-e — Vertical Cylinder, Stream Lines Ra = 51, Re = 0, Nu = 2.09. The scale in the radial direction is 5 times the scale in the vertical direction.

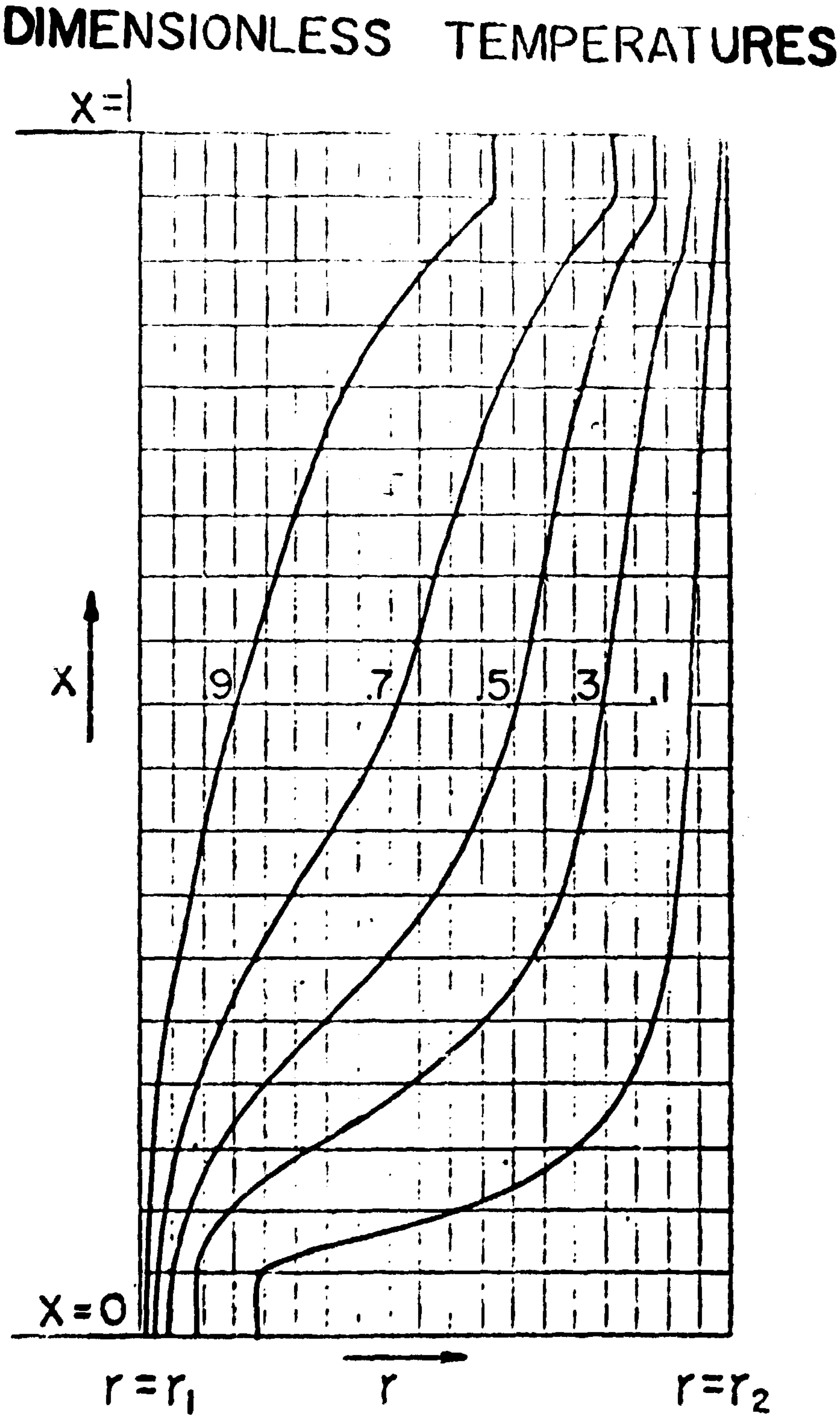


Figure 14-b – Vertical Cylinder, Dimensionless Temperatures Ra = 51, Re = 0, Nu = 2.09. The scale in the radial direction is 5 times the scale in the vertical direction

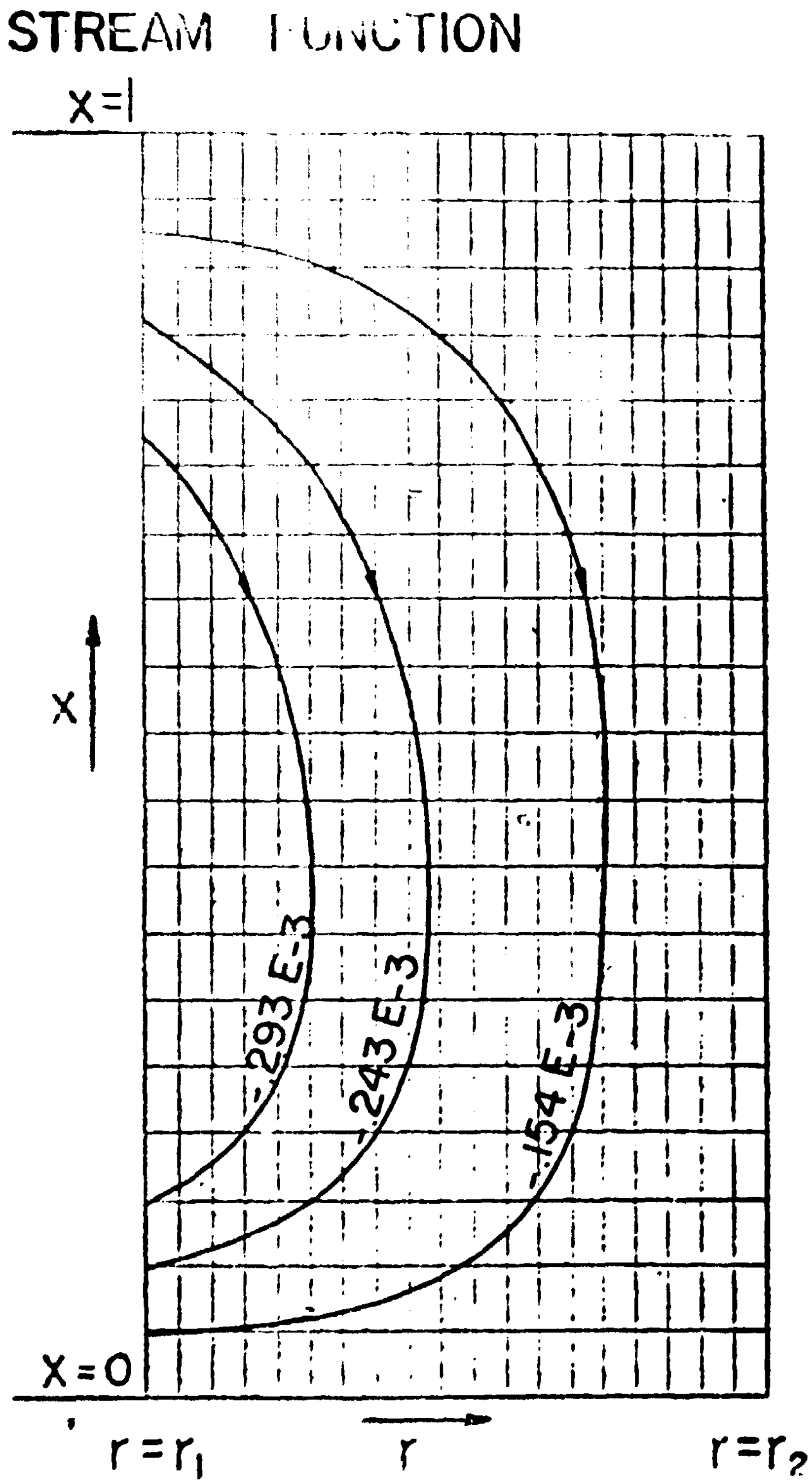


Figure 15-e – Vertical Cylinder, Stream Lines Ra = 5.1, Re = 161, Nu = 4.67. The scale in the radial direction is 5 times the scale in the vertical direction.

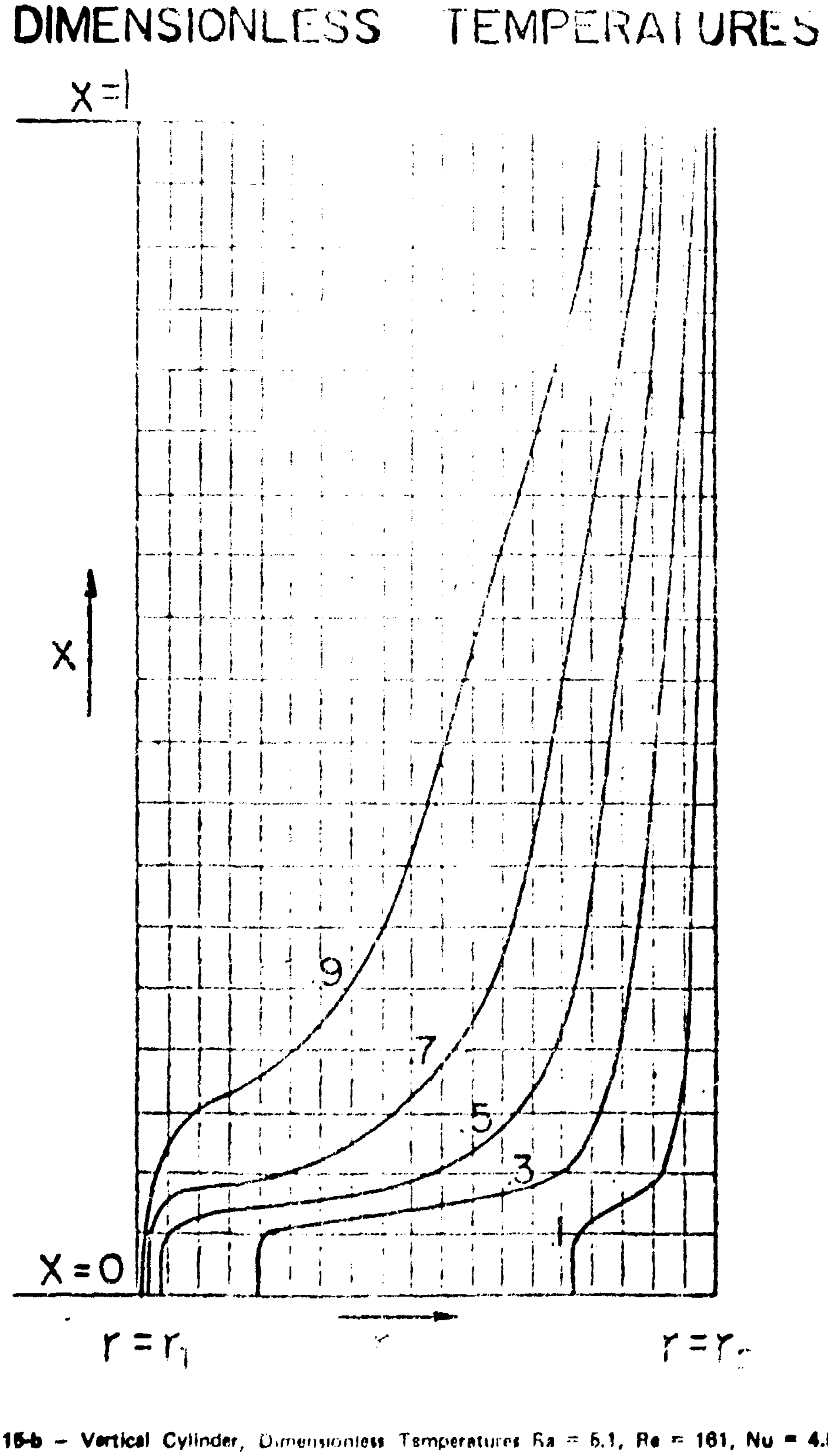


Figure 15-b – Vertical Cylinder, Dimensionless Temperatures Ra = 5.1, Re = 161, Nu = 4.57. The *scale in the radial direction is 5 time the scale in the vertical direction.*

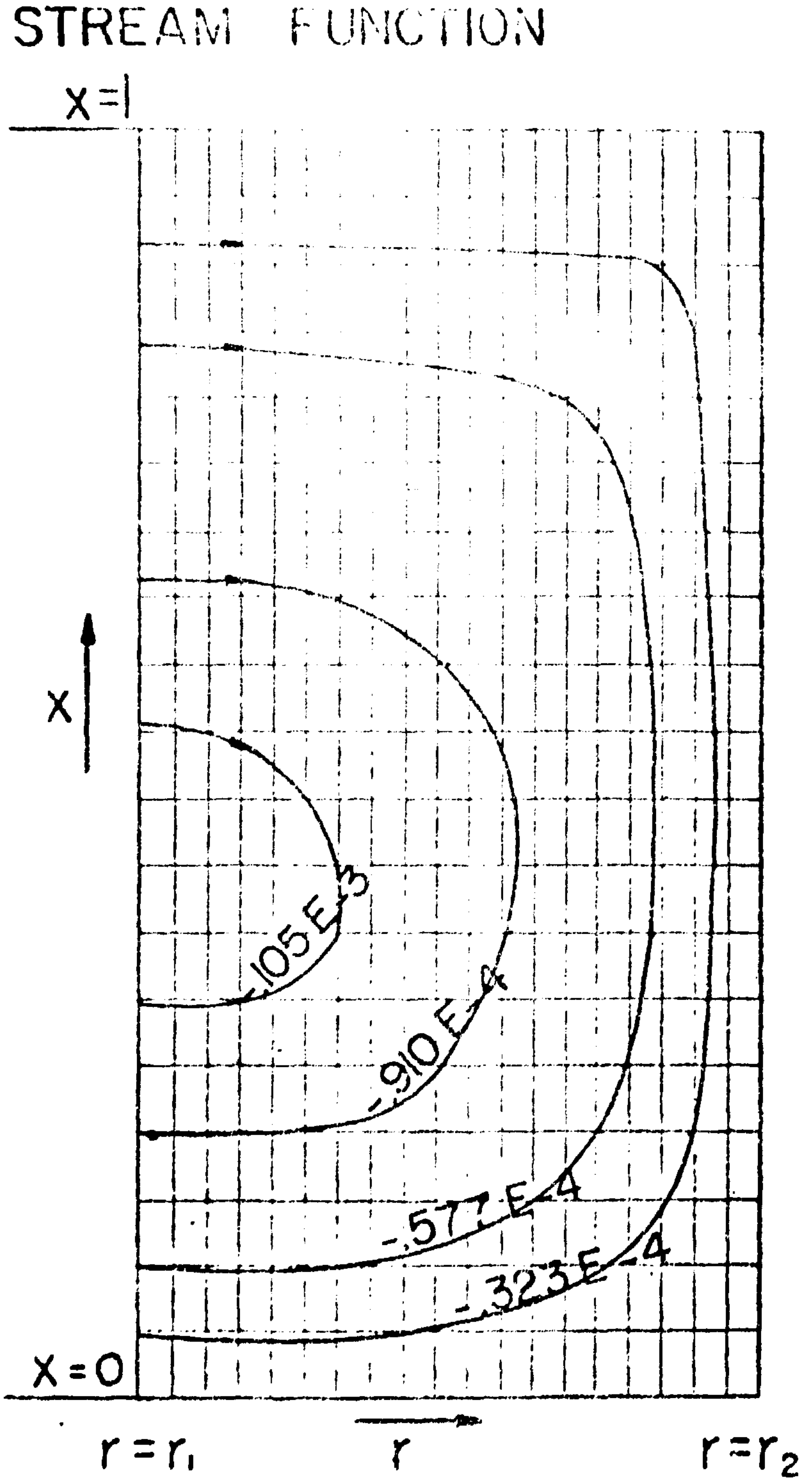


Figure 16-a – Vertical Cylinder, Stream Lines Ra = 61, Re = 29, Nu = 2.68. The scale in the radial direction is 6 times the scale in the vertical direction.

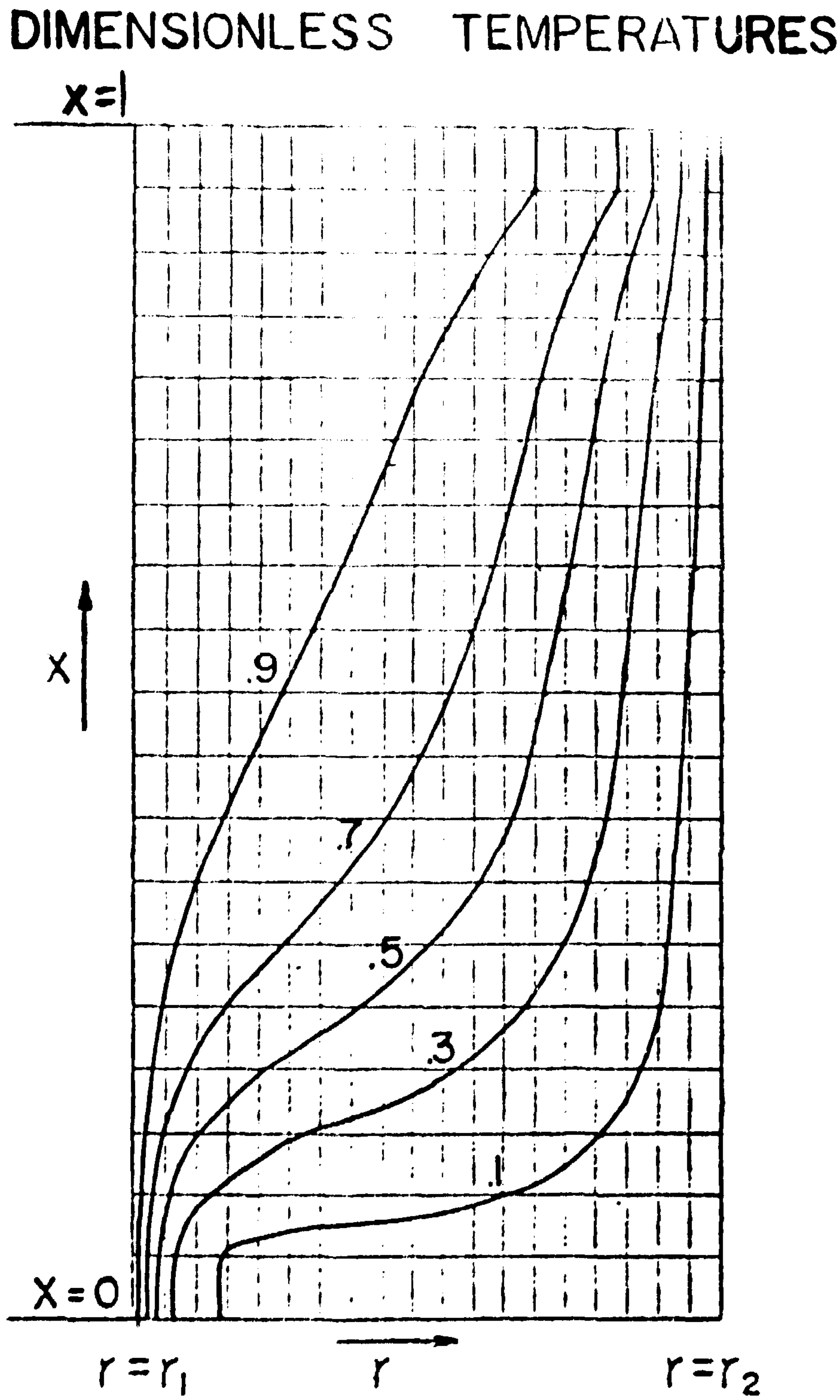


Figure 16-b – Vertical Cylinder, Dimensionless Temperatures Ra = 51, Re = 29, Nu = 2.58. The scale in the radial direction is 5 times the scale in the vertical direction

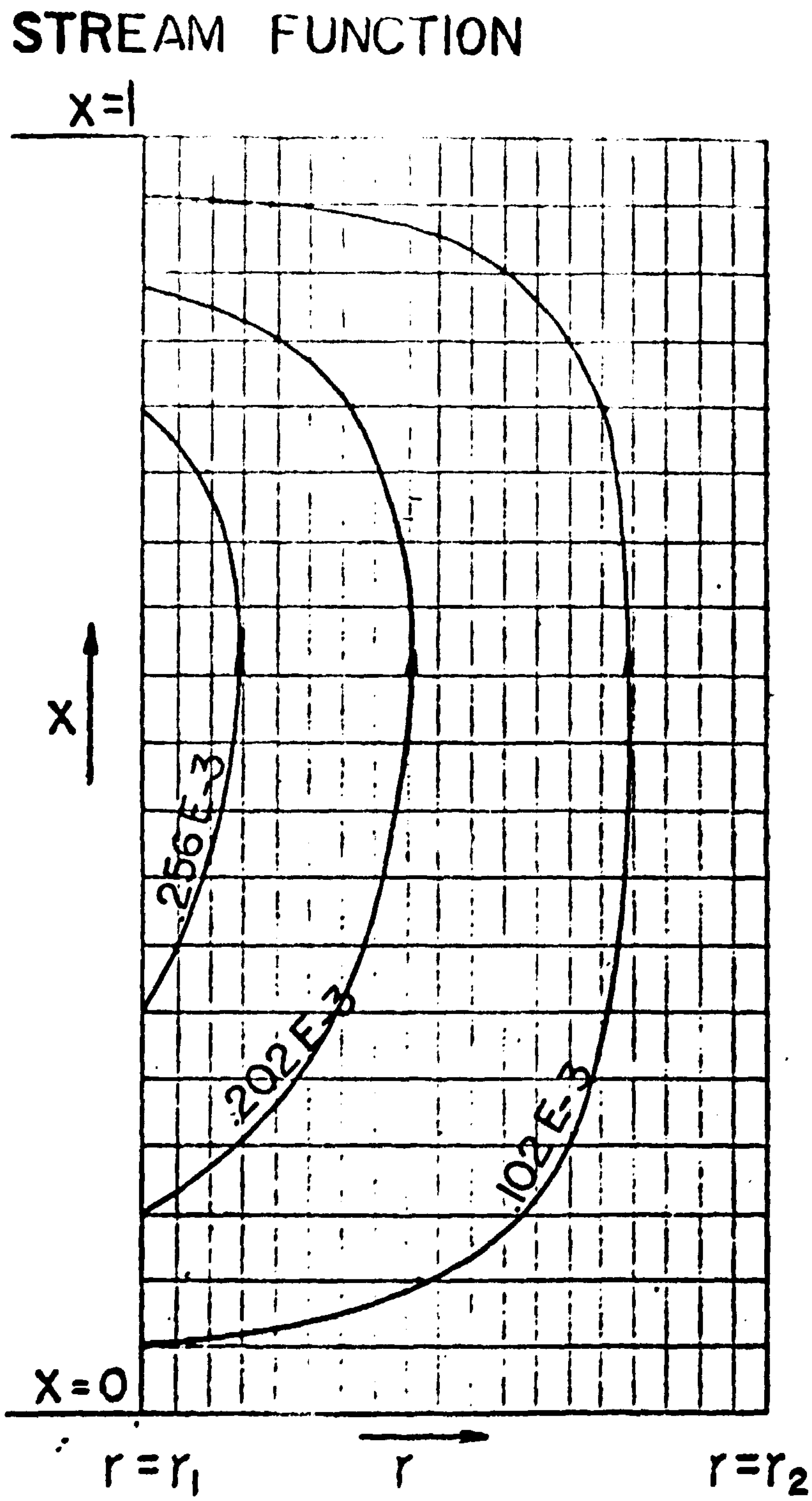


Figure 17-e – Vertical Cylinder, Stream Lines Ra = 5.1, Re = -132, Nu = 3.89. The scale in the radial direction is 5 times the scale in the vertical direction.

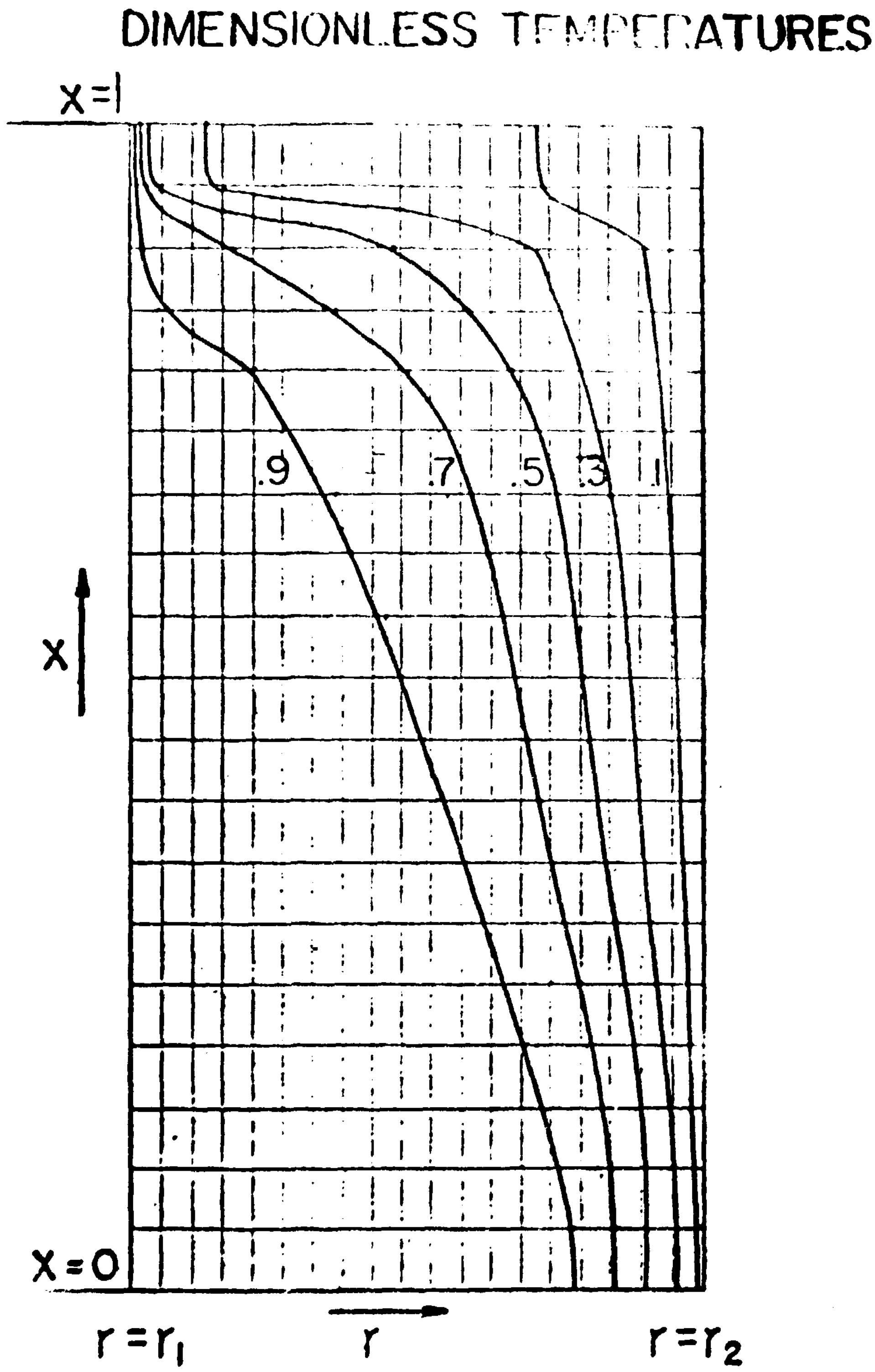


Figure 17-b – Vertical Cylinder, Dimensionless Temperatures Ra = 5.1, Re = - 132, Nu = 3.89. The scale in the radial direction is 5 times the scale in the vertical direction.

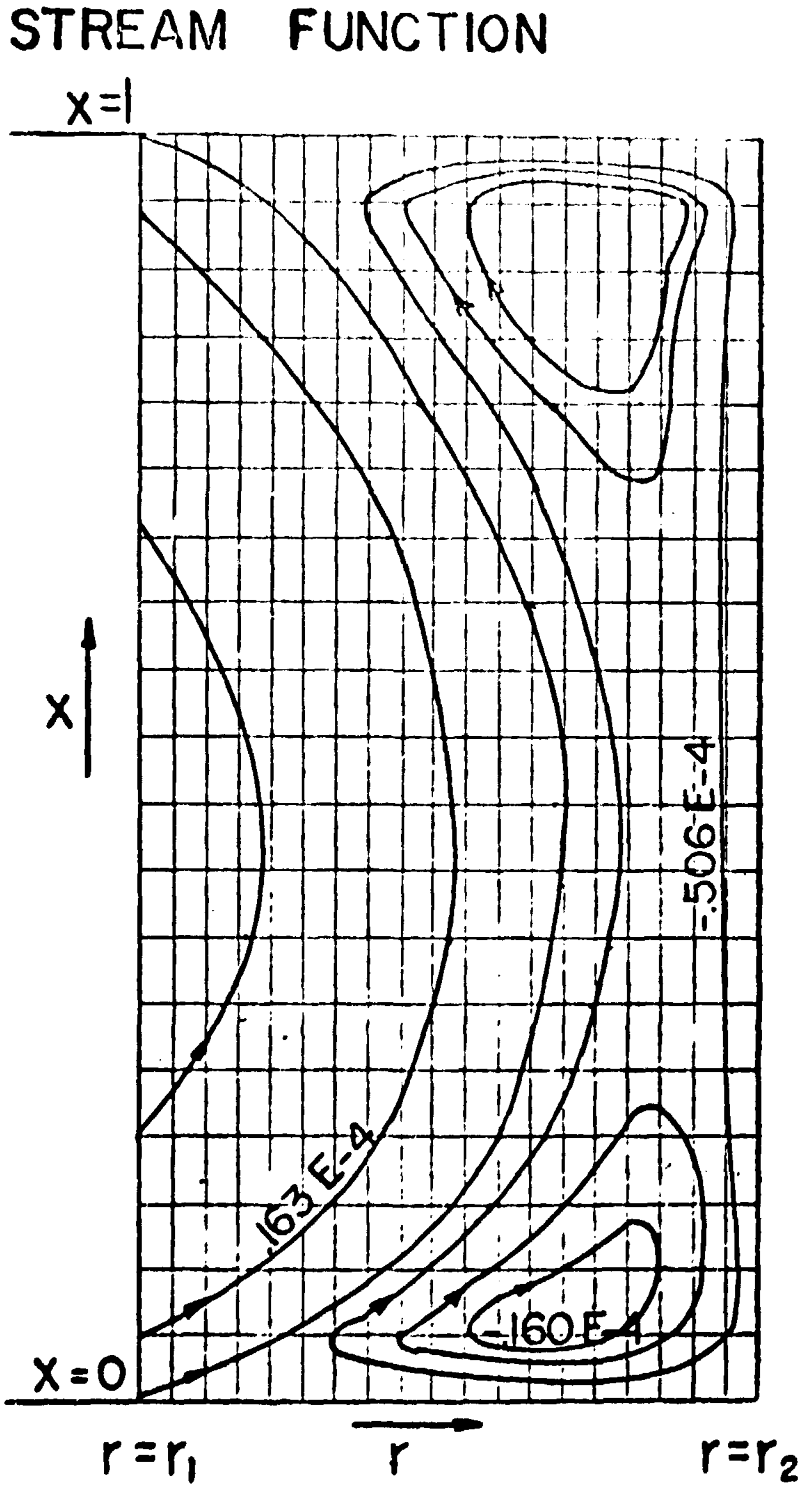


Figure 19-a – Vertical Cylinder, Stream Lines Ra = 51, Re = - 85, Nu = 1,17. The scale in the radial direction is 5 times the scale in the vertical direction.

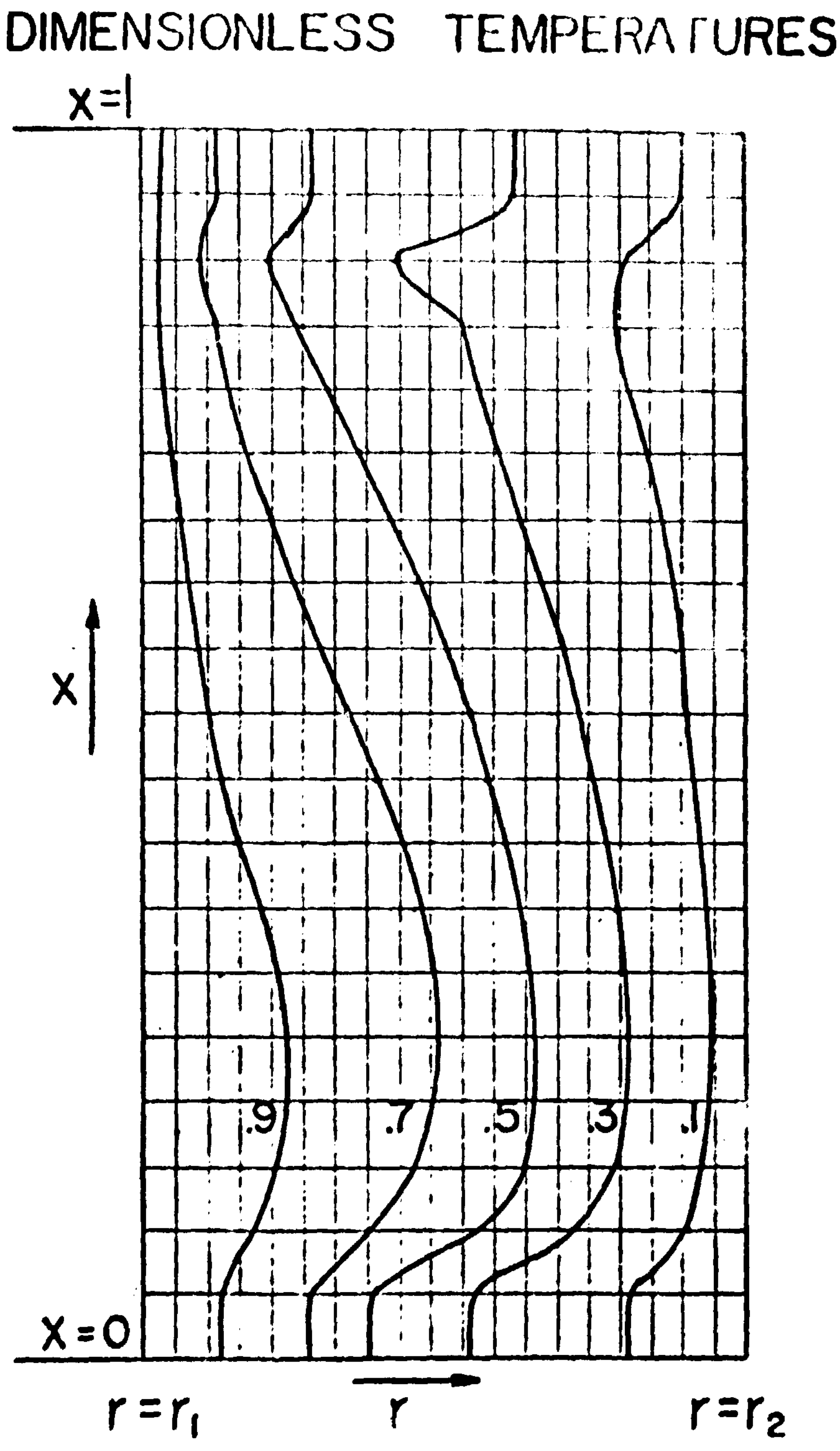


Figure 18-b – Vertical Cylinder, Dimensionless Temperatures Ra = 51, Re = -85, Nu = 1.17. The scale in the radial direction is 5 times the scale in the vertical direction.

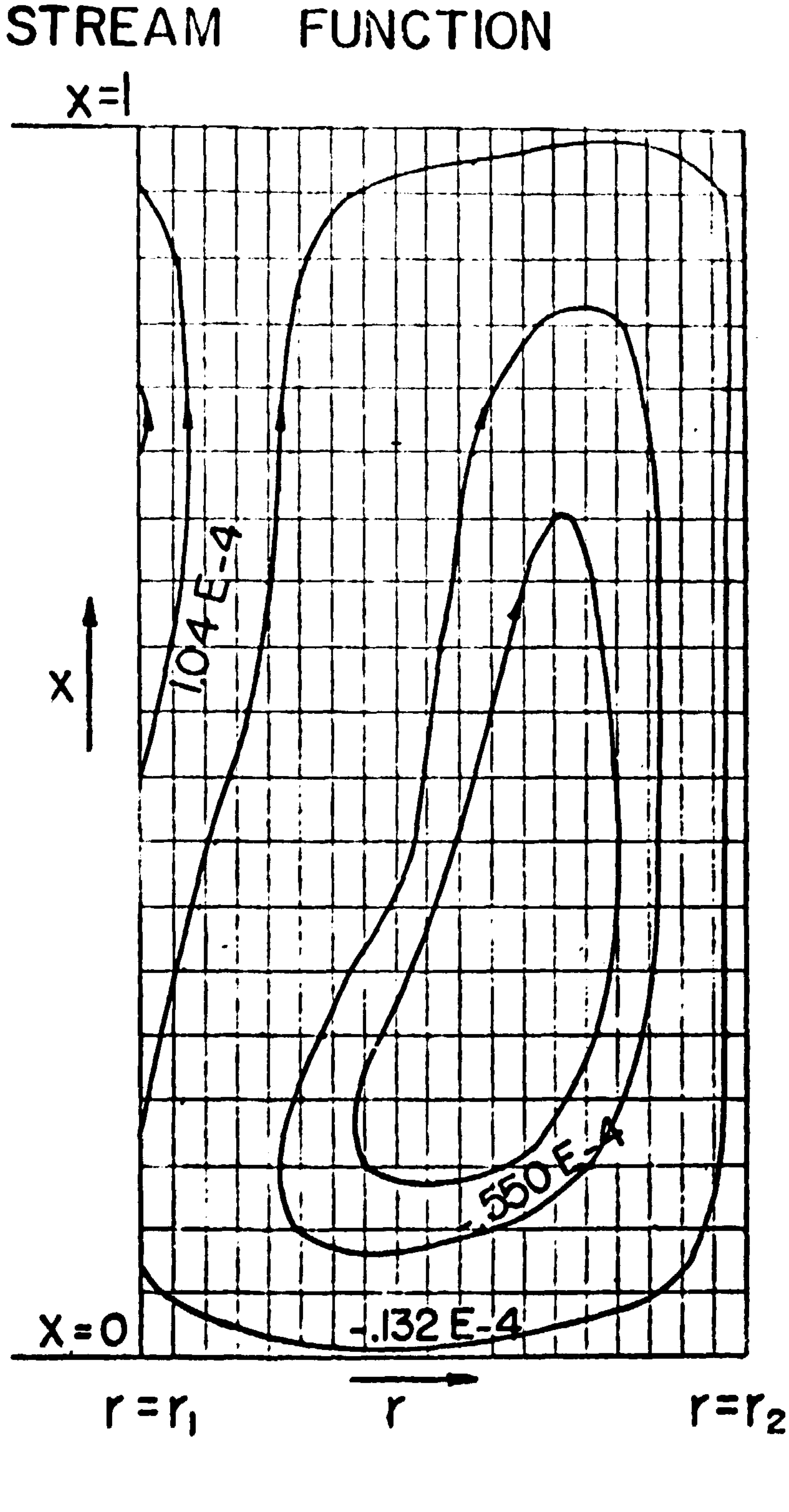


Figure 19-a – Vertical Cylinder, Stream Lines Ra = 128, Re = -128, Nu = 1.62. The scale in the radial direction is 5 times the scale in the vertical direction.

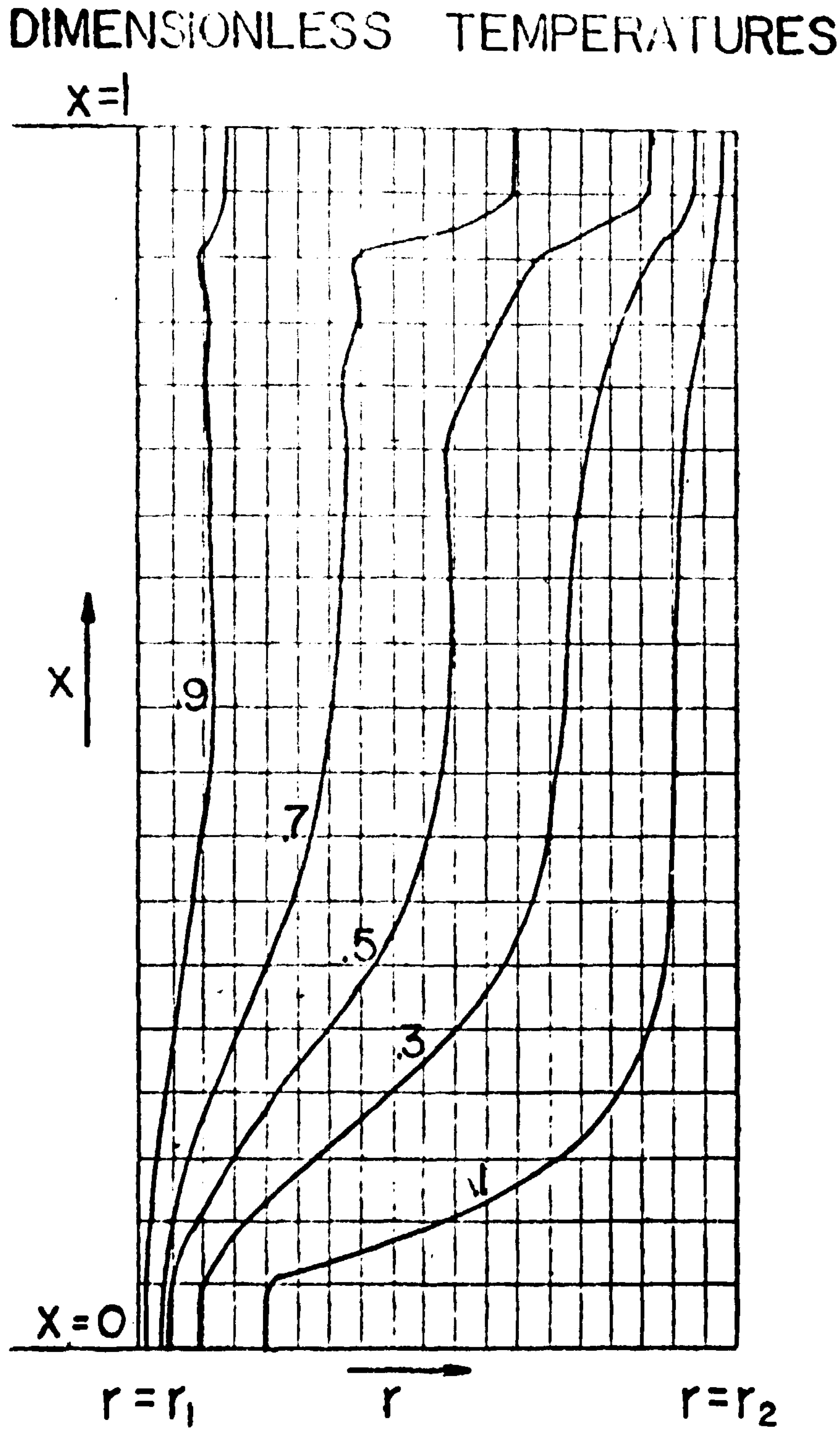


Figure 19-b – Vertical Cylinder, Dimensionless Temperatures Ra = 126, Re = - 128, Nu = 1.62. The scale in the radial direction is 5 times the scale in the vertical direction.

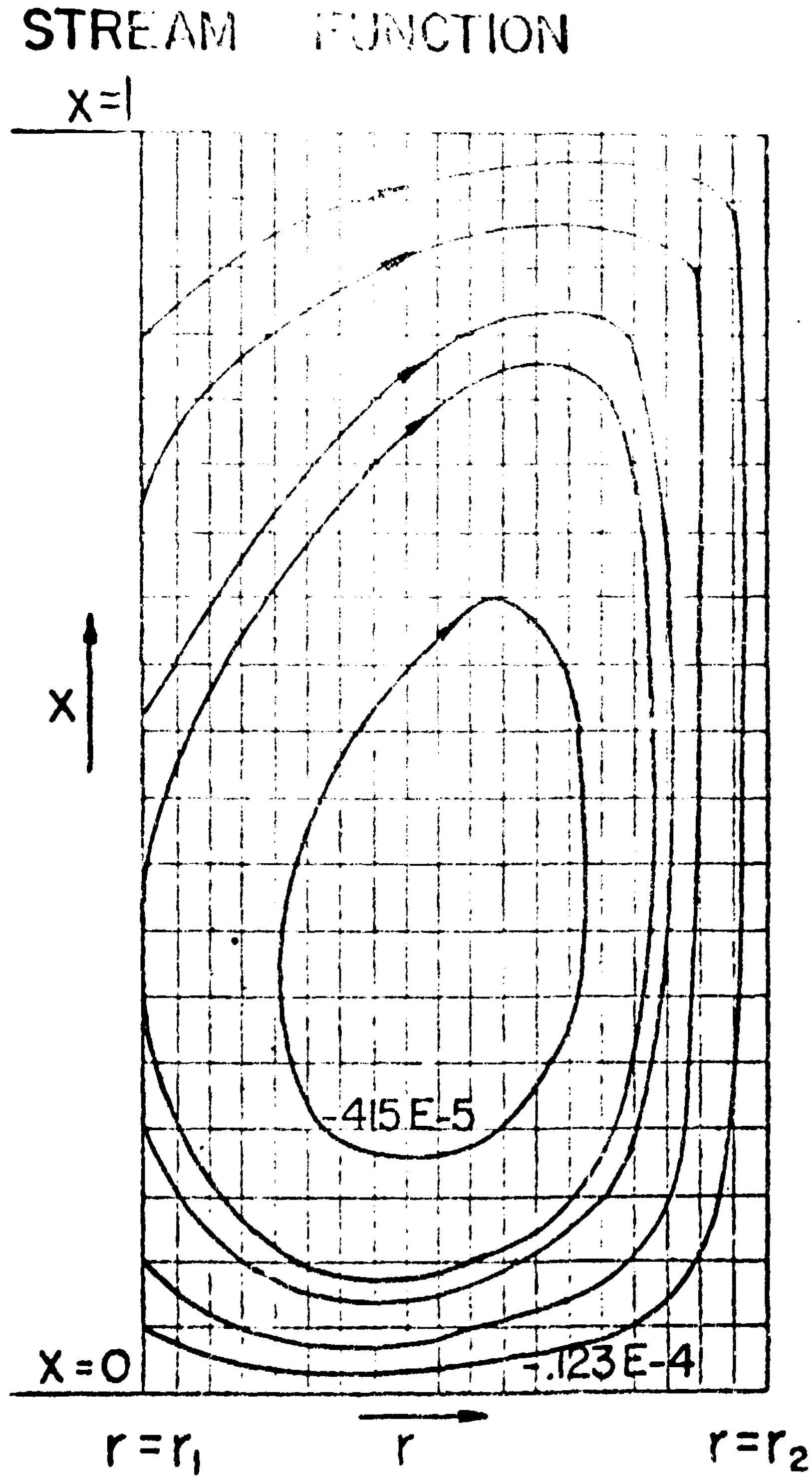


Figure 20-a — Vertical Cylinder, Stream Lines Ra = 51, Re = - 28, Nu = 1.58. The scale in the radial direction is 5 times the scale in the vertical direction.

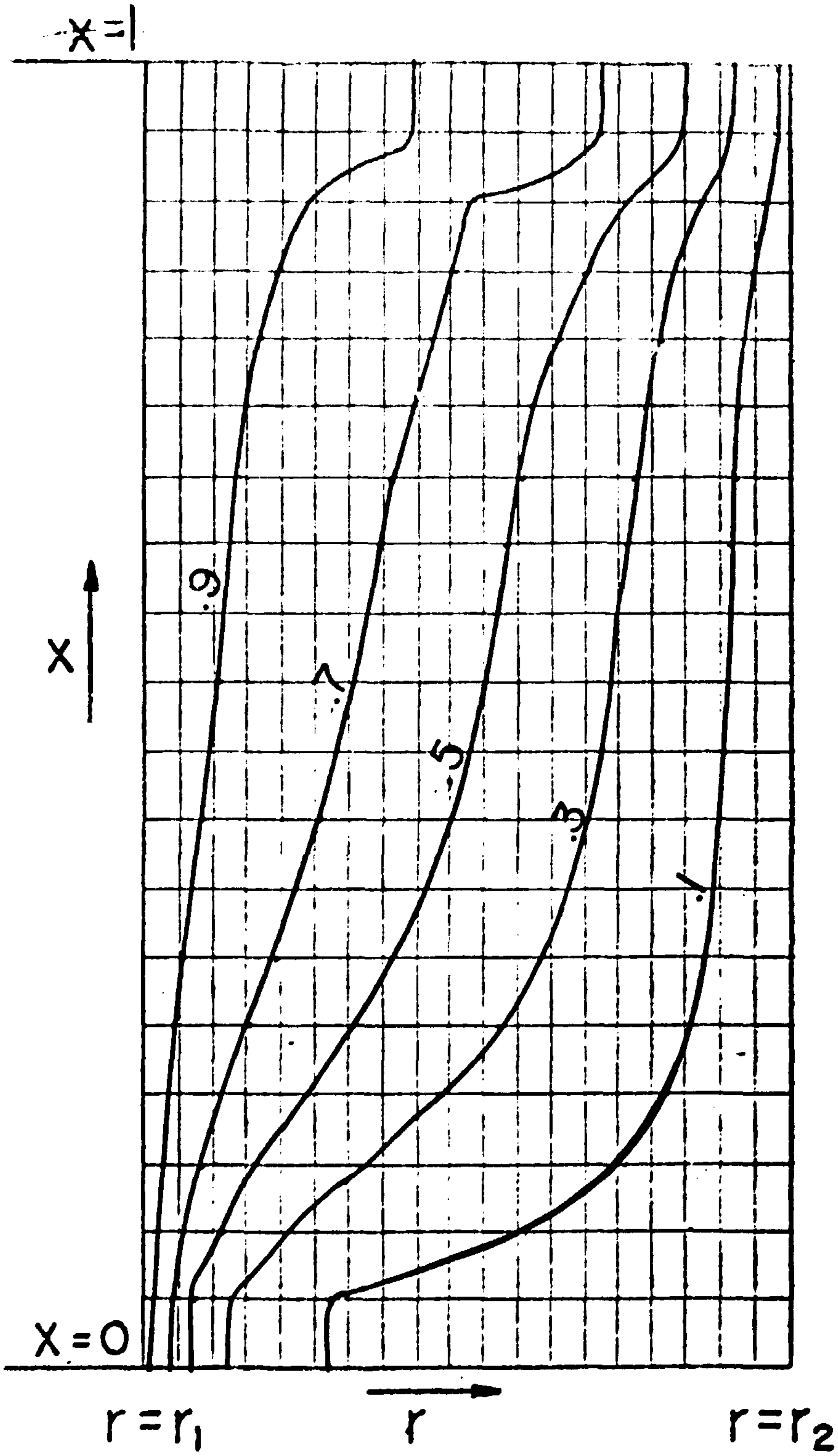


Figure 20-b – Vertical Cylinder, Dimensionless Temperatures Ra = 51, Re = -28, Nu = 1.58. The scale in the radial direction is 5 times the scale in the vertical direction.

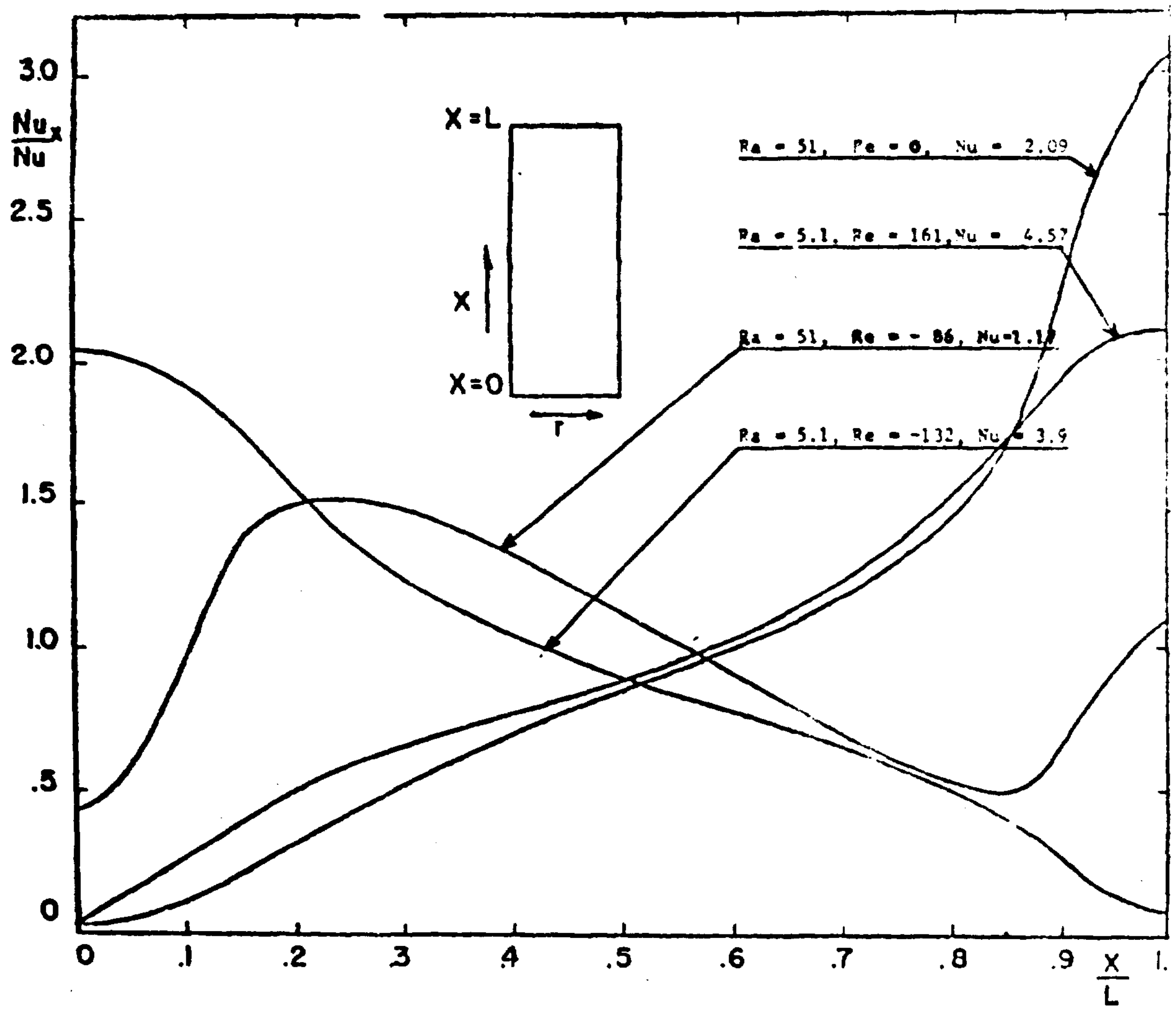


Figure 21 – Vertical Cylinder, Variation of Local Nusselt Number Along the Cold Wall

Correlation of Nu

For a rectangular cavity, Nu has been correlated by Jannot [5] by

$$Nu = .8 \left(\frac{Ra + 1.4\,Pr\,Re}{A} \right)^{.55} \qquad (68)$$

for $4 < .8 \left(\frac{Ra + 1.4Pr\ Re}{A} \right)^{.55} < 10^3$, $A \geq 10$. Where Re can be positive or negative. To obtain Equation (68), the parameter $p = \frac{aK}{dK_c}$ has been set to: 0 and Pr to 1. When p = 0, the resistence of the permeable wall to flow is much less than that of the insulation. Then the pressure gradient in the insulation will be equal to that outside the permeable hot wall. Thus the Reynolds number (Equation (67)) which uses the outside pressure gradient $\frac{\partial p_o}{\partial x}$ will give a good representation of permeation flow in the insulation. On the other hand, when the wall has a relatively high resistence to flow, only a fraction of the outside pressure gradient $\frac{\partial p_o}{\partial x}$ will exist in the insulation. To better approximate the pressure gradient $\frac{\partial p}{\partial x}$ inside the insulation, a combination of Re and p would be necessary. In the present study, Pr has been varied from .4 to .90, and p from 0 to 8. Figure 22 shows a plot of Nu vs $.8 \left(\frac{Ra + 1.4\,Pr\,Re}{A} \right)^{.55}$. The spread of the points indicates the influence of parameters p and Pr. The upper values of Nu are from runs with p between 0 and .4, and lower values from runs with p between 4 and 8. On the other hand runs with constant p and varying Pr indicated that Nu increases about 10% as Pr increases from .4 to .9.

The parameter p can be combined with Re by defining a new Reynolds number, Re^* by

$$Re^* = \frac{Re}{1 + cp} \qquad (69)$$

where c is a constant.

Thus, when the hot wall has no resistence to flow, p = 0, and $Re^* = Re$. The shielding effect of the high wall resistence would be taken into account by Re^* since Re^* would be smaller as p became larger. Also the influence of the Prandtl number could be better represented by raising it to an expoent. A more general correlation for Nu thus would be of the form

$$Nu = C_1 \left(\frac{Ra + m\,Pr^n\,Ra^*}{A} \right)^{.55} \qquad (70)$$

where c_1, m, and n are constants.

Due to time constraints, numerical values for the constants have not been evaluated. Instead, the Nusselt number have been correlated with

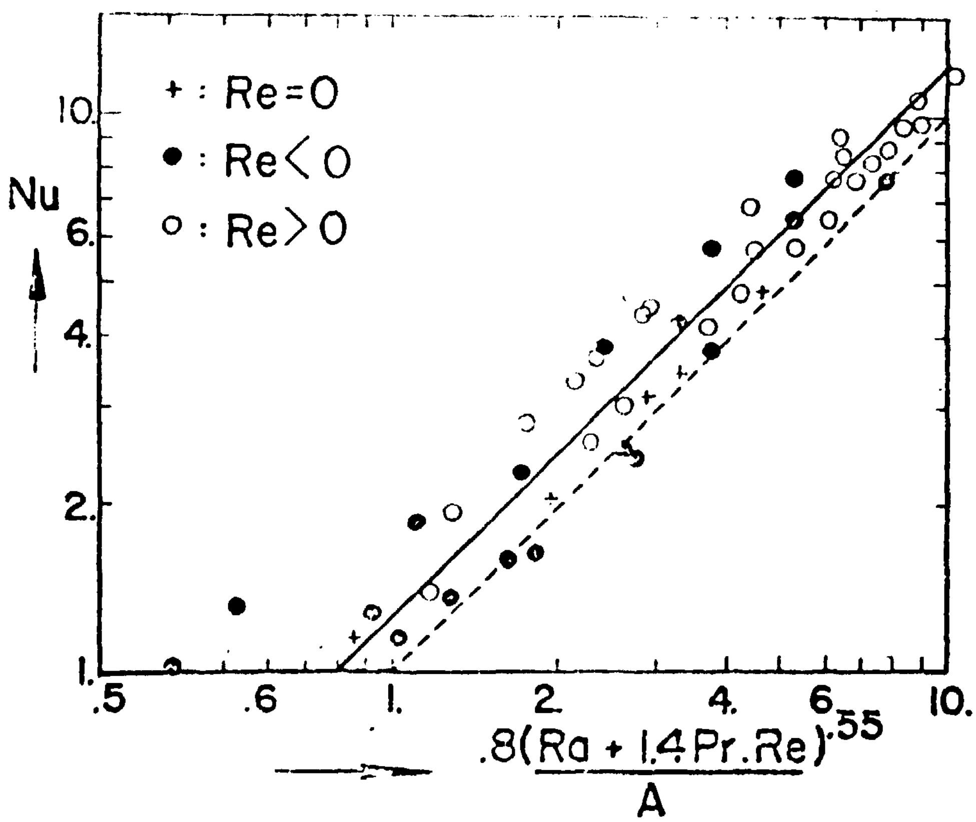


Figure 22 – Vertical Cylinder With Permeation Flow, Correlation of Nusselt Number

$$Nu = .98 \left(\frac{Ra + 1.4\ Pr\ Re}{A} \right)^{.55} \quad (71)$$

$$\text{for } .98 \left(\frac{Ra + 1.4\ Pr\ Re}{A} \right)^{.55} > 1 \text{ and } A \geqslant 10.$$

For values of $A < 10$, Equation (68) does not hold, and Nu becomes a function of A also (Equation (68) predicts higher values for $A < 10$). The validity of Equation (71) is assumed to be for $A \geqslant 10$ also. The computer points are for $20 > A \geqslant 10$.

The error band for the correlation was ± 10%. Some of the spread is due to the non-ideal behaviour of carbon dioxide. For Helium, the computed Nusselt numbers are expected to show a smaller spread.

4.3 – Rectangular Geometries

Rectangular geometries habe been modeled to check the accuracy of the present numerical model. The range pf parameters as well as the computation of the heat fluxes and Nu have been similar to what has been discussed earlier for the cylinders.

Closed Hot Wall

Figure 23 shows the temperature and flow fields for a vertical cavity with closed hot wall. The profiles are close to those for a vertical cylinder (Figure 12).

The correlation of Nu with Ra is shown on Figure 24. The computed points agree very well with the correlation of Jannot[5], obtained from computed results, and also checked with experimental data. The correlation is

$$Nu = .55 \left(\frac{Ra}{A} \right)^{.5} \qquad \text{for } A \geqslant 10 \quad (72)$$

Permeable Hot Wall

To compare the present model with that of Jannot, FINS has been run with the same p ($p \simeq 0$) and same Pr ($Pr \simeq 1$). The results of the flow temperature profiles are not shown since they are similar to those for a vertical cylinder. The case with horizontal walls has not been investigated, but the computer program FINS has been provided with that option.

The computed Nu has been plotted vs $.8 \left(\frac{Ra + 1.4\ Pr\ Re}{A} \right)^{.55}$ (Figure 25). It is seen that there is very good agreement between the results of Jannot and those from the present work for a rectangular cavity.

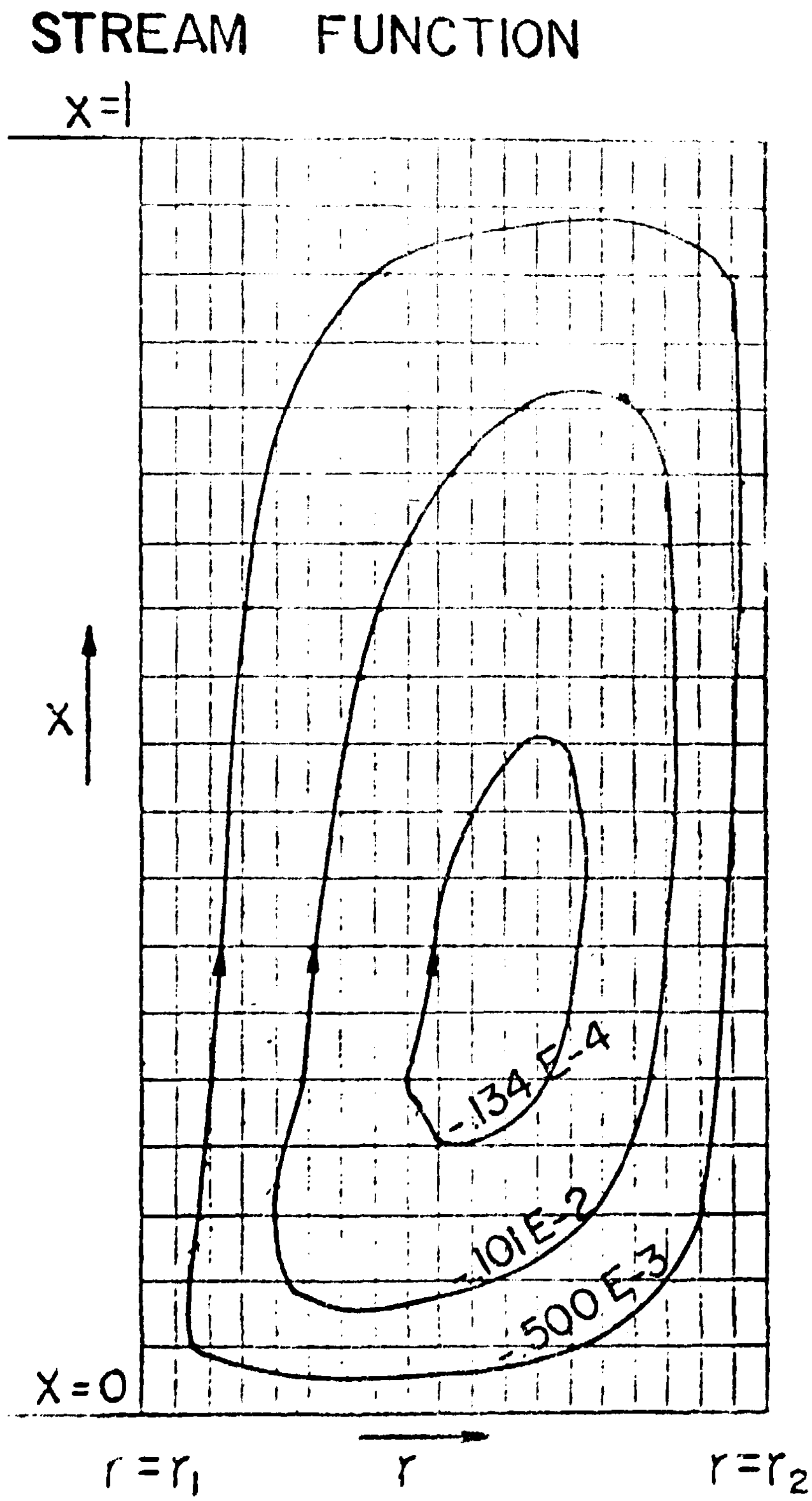


Figure 23-e – Rectangular Geometry Without Permeation Flow, Stream Lines, Ra = 209, Nu = 2.46

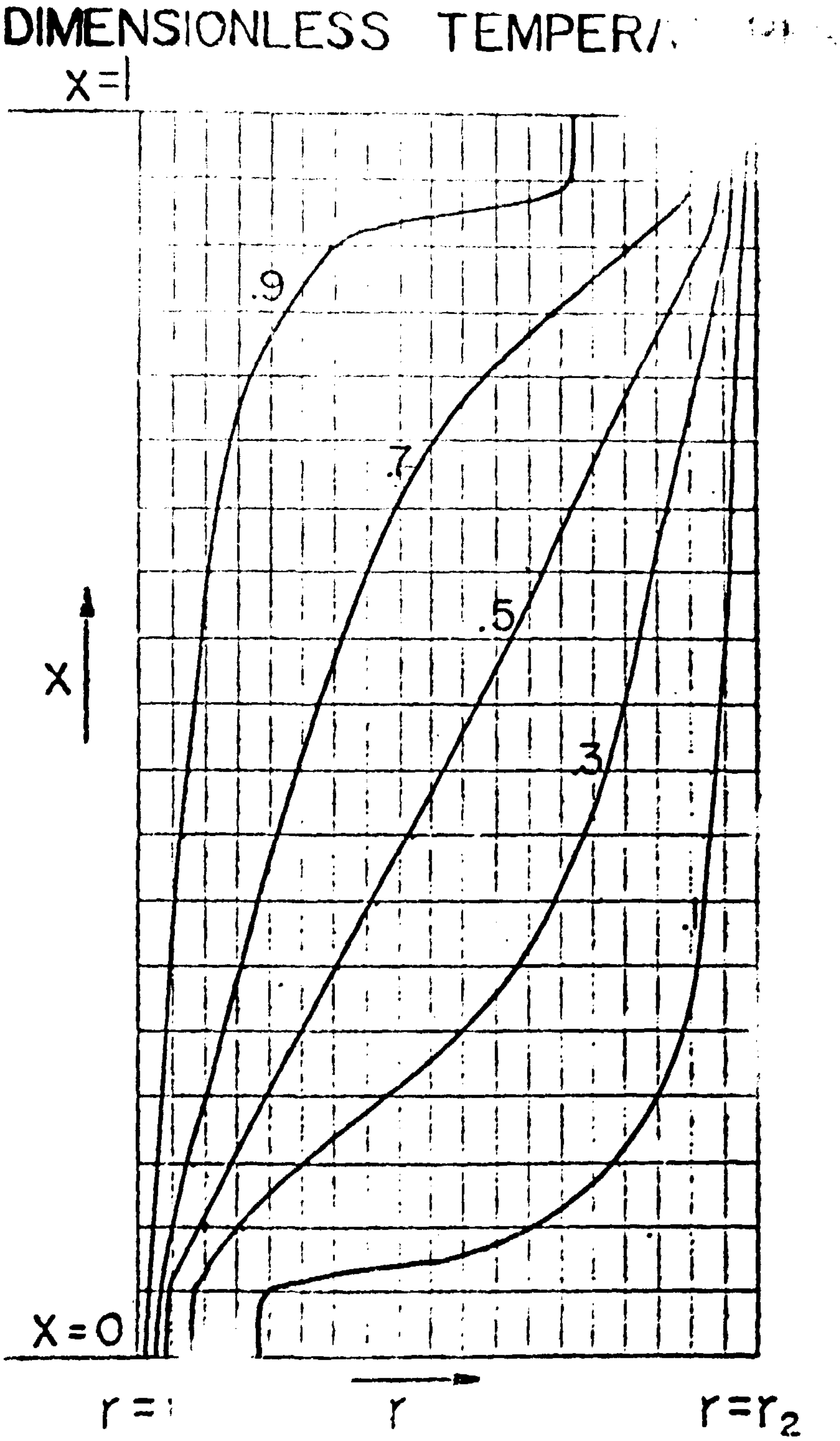


Figure 23-b – Rectangular Geometry Without Permeation Flow, Dimensionless Temperatures, Ra = 209, Nu = 2.46.

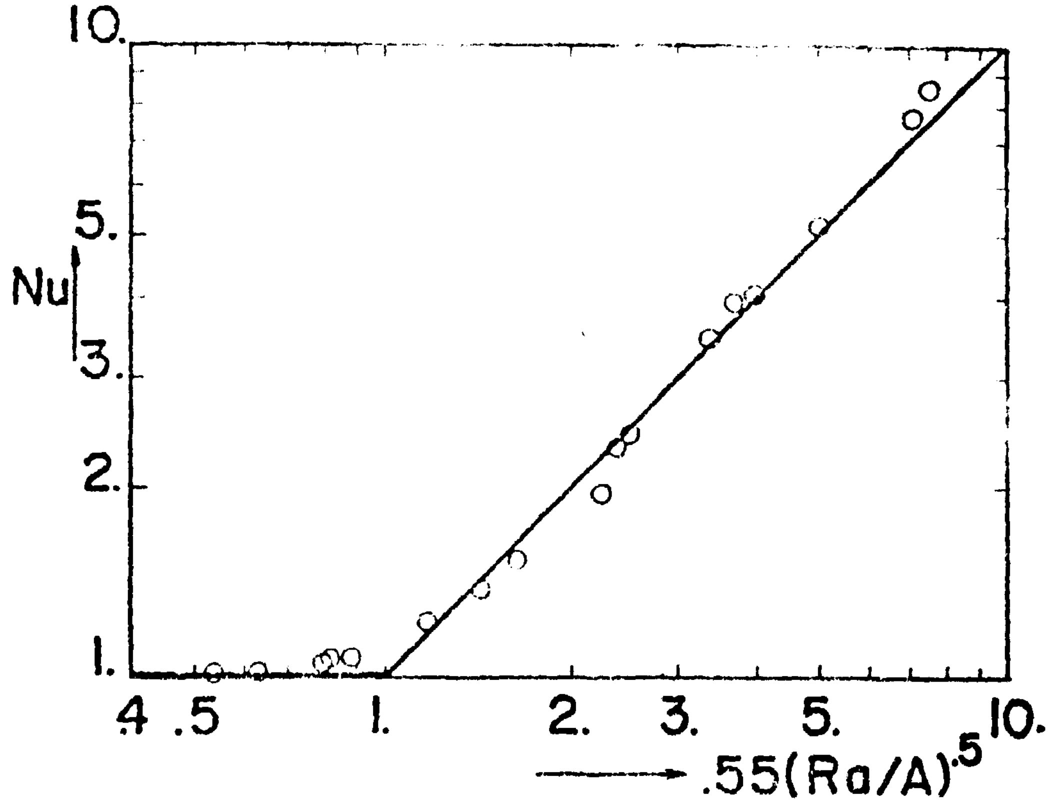


Figure 24 — Correlation of Overall Nusselt Number, Rectangular Geometry Without Permeation Flow

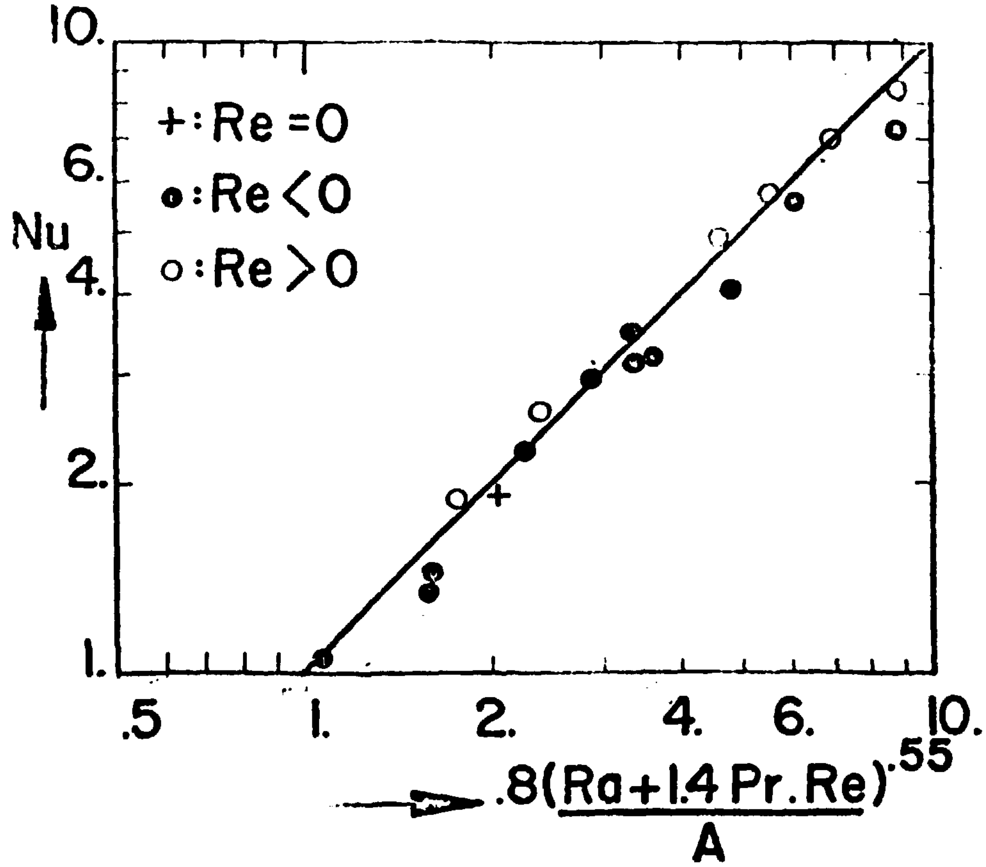


Figure 26 – Correlation of Overall Nusselt Number, Rectangular Geometry, Permeable Hot Wall

5 – CONCLUSION

5.1 – Summary of Results

A numerical solution of flow and heat transfer in fibrous insulation has been obtained by modelling the fibrous insulation as a porous medium. Cylindrical and rectangular geometries with a variety of boundary conditions (vertical or orizontal walls, closed or permeable hot wall, convective boundary conditions or given temperatures at the walls) have been considered. The method allows for variable properties and real gas behaviour through the use of property subroutines.

Excellent agreement has been obtained with published numerical and experimental work on insulation packed between rectangular walls. Comparisons for the cylindrical geometries have not been possible due to the lack of available literature results. However, since the same physical model and the same numerical method have been used for ll geometries, the computations for cylindrical cavities are expected to be valid also.

The details of flow and heat transfer for natural, forced, and a combination of both have been obtained for a vertical cylinder. Forced convection flows are in the same direction (downward along the cold wall) as natural convection flows when the gas outside the hot wall flows downward and in the opposite direction (upward along the cold wall) when it flows upward. The local Nusselt number along the cold wall increases from bottom to top for natural convection, forced convection with downward gas flow, or both. When the gas flow upwards, and the natural convection is negligible, Nusselt number decreases from bottom to top.

Average effective Nusselt number for the insulation has been correlated with Rayleigh number only for closed hot wall, and with Rayleigh and Reynolds numbers for permeable hot wall. Forced convection flows can reach very high values when the outside pressure gradient is high. Both natural and forced circulation increase with increasing fiber permeability and gas density. The insulation must be protected from forced convection flows and fibers of small permeability (high packing density, small fibre diameter) must be chosen to obtain good insulation between the hot gas and the cold wall. The numerical model presented provides a useful tool to assess the performance of the fibrous insulation by accounting accuratly for the natural and forced convection flows.

5.2 – Future Work

Future work concerns the application of the numerical method to experimental work on fibrous insulation to be carried out at IEA Helium Loop. The application involves first the incorpotarion in the computer program FINS of the test section geometry and boundary conditions, and second the determination of input parameters to the numerical method. The modification of FINS is not diffult, and no convergence problems should be encountered with the test section geometry. The input parameters which must be determined are the permeability and the effective thermal conductivity of the insulation in the absence of forced and natural convection.

NOMENCLATURE

A — aspect ratio, defined by

$$A = \frac{l}{d} \text{ for a rectangular cavity}$$

$$A = \frac{l}{r_2 - r_1} \text{ for a vertical cylinder}$$

$$A = \frac{\Pi (r_1 + r_2)}{2 (r_2 - r_1)} \text{ for a horizontal cylinder}$$

a — thickness of the cover plate

b — weight parameter for updating stream function

c — weight parameter for updating temperatures

C_p — specific heat of the fluid

d — thickness of the porous medium for a rectangular cavity $d = r_2 - r_1$ for cylindrical geometries

h — heat transfer coefficient

K — permeability of porous medium

l, L — length of the vertical cylinder or rectangular cavity

Nu — Nusselt number $= \frac{hd}{\lambda_r}$

p — pressure

Pr — Prandtl number $= \frac{\mu C_p}{\lambda}$

q heat flux

r radius, radial cylindrical coordinate

Ra Rayleigh number, defined by

$$Ra = \frac{g(T_H - T_c)\ dk\ \beta}{\nu_r \alpha_r}$$

Where $K = K_r$ for cylindrical and $K = K_y$ for rectangular geometries

Re Reynolds number, defined by

$$Re = \frac{K_x(\partial p_o/\partial x - \rho_o g_x)\, d}{\mu_r \nu_o}$$

T temperature

v velocity

x axial cylindrical coordinate or cartesian coordinate parallel to the cover plate

y cartesian coordinate perpendicular to cover plate

α thermal diffusivity $= \frac{\lambda}{\rho C_v}$

β thermal coefficient of volumetric expansion at constant pressure, defined by

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)p$$

θ angular cylindrical coordinate

λ thermal conductivity of the porous medium with stationary fluid

μ dynamic viscosity of the fluid

ν kinematic viscosity of the fluid

ρ	density of the fluid
ψ	stream function

Subscripts

C	cover plate properties
n	itiration number
o	free stream quantities
	quantities calculated at reference condition
θ,r,x,y	porperties in θ, r, x, and y directions

Superscripts

	node index in r or y directions
	node index in x direction

APPENDIX

COMPUTER PROGRAM FINS
INPUT INFORMATION AND LISTING

Input Information

1) Permeability K (PER in FINS) depends on the fiber diameter, orientation of the fibers, and the insulation porosity. Furber and Davidson[3] give their experimental results and an empirical correlation of K with the fiber diameter and insulation porosity. The correlation is not very accuracy because it does not account for the fiber orientation. For more accurate determination of K, experimental data on the particular insulation to be modeled is required. The insulation must be packed in the apparatus for K measurement the same way as it is packed in the test section to be simulated by FINS.

2) Effective thermal conductivity λ (AKO in FINS) of the insulation with stagnant gas.

   λ has been modeled as

   $$\lambda = \epsilon\lambda_{gas} + \lambda_{fiber} \qquad \text{(AKO = POR * AK + AKO)}$$

   λ_{fiber} (AKO) must be given either from a correlation of from experimental data. It depends on the porosity, fiber diameter fiber orientation, and the thermal conductivity of the fiber material. Thermal conductivity experiments under vacuum yield λ_{fiber}. An approximation to λ_{fiber} has been obtained from available literature data as

   $$\lambda_f = 6.0\text{ E-5 } d \qquad \text{(W/ms)}$$

   where d is the insulation density (Kg/m^3).

3) Permeability of the hot wall, K_c (PERCEP in FINS) information on the evaluation of K_c can be found in reference[5].

4) Pressure drop in the gas stream $\frac{\partial p_o}{\partial x}$ (DPDX in FINS). This can be calculated from:

   $$\frac{\partial p_o}{\partial x} = \frac{f \rho v^2}{D_H \; 2 g_c}$$

   where f is the friction factors, ρ gas density, and v gas velocity.

```
COMPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=60,SIZE=0000K,
                   SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF
      C      FINS (FIBROUS INSULATION)     WRITTEN BY AYDIN KONUK,
      C           INSTITUTO DE ENERGIA ATOMICA', SAO PAULO, BRAZIL
      C           PRESENT VERSION COMPLETED IN MAY 15, 1976.
      C
      C      PURPOSE
      C           TO OBTAIN A NUMERICAL SOLUTION OF TEMPERATURE AND
      C           VELOCITY DISTRIBUTION IN FIBROUS ISOLATION PACKED
      C           IN ANNULAR OR RECTANGULAR CAVITIES.
      C
      C      DESCRIPTION INPUT PARAMETERS
      C
      C         GEOMETRY AND BOUNDARY CONDITIONS
      C           NG=1     -HORIZONTAL CYLINDER
      C           NG=2     -VERTICAL CYLINDER
      C           NG=3     -RECTANGULAR GEOMETRY, VERTICAL WALLS
      C           NG=4     -RECTANGULAR GEOMETRY, HORIZONTAL WALLS,
      C                     WITH HOT WALL ON TOP
      C           NG=5     -RECTANGULAR GEOMETRY, HORIZONTAL WALLS,
      C                     WITH COLD WALL ON TOP
      C           MBC=0    -GIVEN COLD WALL TEMPERATURE T2
      C           MBC=1    -GIVEN OUTSIDE TEMPERATURE TOUT AND HEAT
      C                     TRANSFER COEFFICIENT AGAC
      C           NBC=0    -GIVEN HOT WALL TEMPERATURE T1
      C           NBC=1    -GIVEN HOT GAS TEMPERATURE TGAS AND HEAT
      C                     TRANSFER COEFFICIENT AGAH.
      C           NPF=0    -CLOSED HOT WALL
      C           NPF=1    -PERMEABLE HOT WALL, WITH PERMEABILITY
      C                     PERCP AND THICKNESS TCP
      C           R1       -INSIDE RADIUS. R1=0 FOR RECTANGULAR
      C                     GEOMETRIES. IN M.
      C           R2       -OUTSIDE RADIUS. R2=INSULATION THICKNESS
      C                     FOR RECTANGULAR GEOMETRIES. IN M.
      C           AL       -LENTH OF THE INSULATION. FOR HORIZONTAL
      C                     CYLINDER, AL=3.1416. IN M.
      C           PER      -PERMEABILITY OF THE INSULATION, M**2
      C           GR,GTH   -FOR A RECTANGULAR GEOMETRY, COMPONENTS
      C                     OF THE ACCELERATION OF GRAVITY G.
      C                     GR=0, GTH=-G FOR VERTICAL WALLS
      C                     GR=1., GTH=.0 FOR HORIZONTAL WALLS,
      C                     WITH HOT WALL ON TOP,
      C                     GR=-1., GTH=.0 FOR HORIZONTAL WALLS,
      C                     WITH COLD WALL ON TOP.
      C
      C         INSULATION PROPERTIES
      C           AKM      -CONDUCTION THROUGH FIBER
      C           POR      -POROSITY OF THE INSULATION
      C           AKO      -EFFECTIVE THERMAL CONDUCTIVITY OF THE
      C                     INSULATION WHEN THE FLUID IS STAGNANT.
      C           DPDX     -PRESSURE GRADIENT IN THE GAS STREAM,
      C                     N/M**3
      C
      C           PO       -PRESSURE OF THE GAS, BAR
      C           V        -KINETIC VISCOSITY OF THE GAS, M**2/S
      C           AK       -THERMAL CONDUCTIVITY OF THE GAS, W/(M*K)
      C           CP       -HEAT CAPACITY OF THE GAS, W*S/(KG*K)
      C           RO       -DENSITY OF THE GAS, KG/M**3
```

```
C
C     DESCRIPTION OF OUTPUT PARAMETERS
C           J        -NODE INDEX ALONG THE LENGTH OF THE
C                     INSULATION. J=1 AT THE BOTTOM
C                                 J=JJ ON TOP
C           I        -NODE INDEX ACROSS THE THICKNESS OF THE
C                     INSULATION. I=1 AT THE HOT WALL
C                                 I=II AT THE COLD WALL
C
C           TR,TEND  -TEMPERATURES FROM THE SOLUTION OF THE
C                     ENERGY EQUATION,K
C           T,TLINE  -TEMPERATURES USED TO LINEARIZE THE
C                     EQUATIONS OF MOMENTUM AND ENERGY.
C           SR,SEND  -STREAM FUNCTION FROM THE SOLUTION.
C                     OF THE MOMENTUM EQUATION.
C           S,SLINE  -STREAM FUNCTION USET TO LINEARIZE
C                     THE ENERGY EQUATION.
C           TEMER    -TEND-T AT THE LAST ITIRATION.
C           MVR      -RADIAL COMPONENT OF MASS VELOCITY,KG/(S*M**2)
C                     MVR IS POSITIVE GOING FROM THE HOT TO THE
C                     COLD WALL
C           MVTH     -COMPONENT OF MASS VELOCITY ALONG THE WALLS.
C                     POSITIVE GOING UP FOR VERTICAL WALLS.
C           AKEF     -EFFECTIVE THERMAL CONDUCTIVITY AT EACH
C                     J. AT THE WALLS, THE TEMPERATURE
C                     GRADIENT IS APPROXIMATED WITH LINEAR
C                     TEMPERATURE PROFILES.
C           AKEFH    -EFFECTIVE THERMAL CONDUCTIVITY USING
C                     THE HEAT FLUX AT THE HOT WALL FROM
C                     PARABOLIC APPROXIMATION OF THE
C                     TEMPERATURE PROFILE.
C           AKEFC    -SIMILAT TO AKEFH, BUT AT THE COLD WALL.
C
C           AKHC     -ARITHMETIC AVERAGE OF AKEFH AND AKEFC.
C           RNUS     NUSSEL NUMBER, RNUS=AKHC/AKO.
C           RA       RAYLEIGH NUMBER FOR THE INSULATION.
C           RE       REYNOLDS NUMBER FOR THE INSULATION.
C           PR       PRANDTL  NUMBER FOR THE INSULATION
C
C        NUMERICAL SOLUTION PARAMETERS
C           II       -NUMBER OF NODES ACROSS THE THICNESS
C                     OF THE INSULATION
C           JJ       -NUMBER OF NODES ALONG THE WALLS
C           TW1,TW2  -WEIGHT FACTORS TO UPDATE TEMPERATURES
C           SW1,SW2  - WEIGHT FACTORS TO UPDATE STREAM FUNCTION
C           KMAX     -MAXIMUM NUBER OF ITIRATIONS
C           EPT      -CONVERGENCE CRITERION, K
C
C     SUBROUTINES AND FUNCTION SUBPROGRAMS USED ARE
C         SUBROUTINE GELB OF IBM SSP TO SOLVE THE SYSTEM OF
C         SIMULTANEOUS LINEAR EQUTIONS
C         FUNCTIONS TO CALCULATE GAS PROPERTIES
C         FUNCTION ROF(P,T) TO COMPUTE DENSITY
C         FUNCTION CPF(P,T) TO COMPUTE SPECIFIC HEAT
C         FUNCTION CLAMF(P,T) TO COMPUTE THERMAL CONDUCTIVITY
C         FUNCTION ETAF(P,T) TO COMPUTE DYNAMIC VISCOSITY
C
C     DIMENSIONS
```

```
                C       A =  (II*JJ*(2*II+1)-(II*(II+1))/2)
                C       SR,TR,SLINE,TLINE,TDIM = (II*JJ)
                C       S,T,SEND,TEND,TTDIM,RO,V,CP,AK = (II,JJ)
                C       R,QT = (II) , TH(JJ) , QPC,QPH(JJ-1)
                C       MVR,Q,CONV,COND = (II,JJ-1) , MVTH = (II-1,JJ)
                C
                C
                C       DIMENSIONS FOR 20*20 MATRIX
ISN 0002                REAL*8 MVR(20,19),MVTH(19,20)
ISN 0003                DIMENSION CGR(5),CGTH(5)
ISN 0004                DIMENSION TDIM(400)
ISN 0005                DIMENSION TTDIM(20,20)
ISN 0006                DIMENSION R(20),TH(20),V(20,20),CP(20,20),AK(20,20)
ISN 0007                DIMENSION RO(20,20)
ISN 0008                DIMENSION SR(400),TR(400),A(16190),TLINE(400),T(20,20)
ISN 0009                DIMENSION SEND(20,20),TEND(20,20),SLINE(400),S(20,20)
ISN 0010                DIMENSION QT(20),Q(20,19),COND(20,19),CONV(20,19),QPH(19)
ISN 0011                DIMENSION QPC(19)
ISN 0012                EQUIVALENCE(TDIM(1),TTDIM(1,1))
ISN 0013                EQUIVALENCE (SLINE(1),S(1,1)),(TLINE(1),T(1,1))
ISN 0014                EQUIVALENCE (SR(1)),SEND(1,1)),(TR(1),TEND(1,1))
ISN 0015                INTEGER R [illegible]
ISN 0016                DOUBLE PRECISION SR,TR,A
ISN 0017                DOUBLE PRECISION SEND,TEND
ISN 0018                DOUBLE PRECISION Q,QT,CONV,COND,AKEF
ISN 0019                DATA CGR,CGTH /[illegible],1.,[illegible],1.,-1.,1.,1.,-1.,.0,.0/
ISN 0020                READ(5,300) NG,NBC,N[illegible],II,JJ,KMAX,EPT
ISN 0021                READ(5,301)R1,R2,AL
ISN 0022                READ(5,302)RO,T1,T2
ISN 0023                READ(5,303)R[illegible]AKR,PR
ISN 0024                IF(NF.EQ.1) READ(5,304) TCP,PERCP,DPDX
ISN 0026                IF(NBC.EQ.1) READ(5,305) TOUT,AGAC
ISN 0028                IF(NBC.EQ.1) READ(5,305) TGAS,AGAH
                C
ISN 0030                G=9.81
ISN 0031                GR=CGR(NG)*G
ISN 0032                GTH=CGTH(NG)*[illegible]
                C
ISN 0033                III=II-1
ISN 0034                JJ1=JJ-1
                C       CALCULATE DR,DR2,DTH,DTH2
ISN 0035                DR=(R2-R1)/III
ISN 0036                DTH=AL/JJ1
                C       CALCULATE R(I),TH(J)
ISN 0037                R(1)=R1
ISN 0038                R(II)=R2
ISN 0039                DO 20 I=2,III
ISN 0040             20 R(I)=R(I-1)+DR
ISN 0041                TH(1)=.0
ISN 0042                TH(JJ)=AL
ISN 0043                DO 21 J=2,JJ1
ISN 0044             21 TH(J)=TH(J-1)+DTH
                C
                C
ISN 0045                DR2=DR*DR
ISN 0046                DTH2=DTH*DTH
ISN 0047                C1=TCP*PERCP/(2.*DTH2*PERCP)
                C
```

```
ISN 0048          M=II*JJ
ISN 0049          N=1
ISN 0050          MUO=II
ISN 0051          EPS=10.**(-5)
ISN 0052          MLO=II
ISN 0053          ME=M*(1+MUO+MLO)- II*(1+II)
            C     APO IS THE PRESSURE IN ATM
ISN 0054          APO=.98692*PO
            C     CALCULATE INITIAL TEMPERATUPF DISTRIBUTION
ISN 0055          DO 30 L=1,M
ISN 0056          TLINE(L)=(T1+T2)/2.
ISN 0057       30 CONTINUE
            C
ISN 0058          KOUNT=0
ISN 0059          TB=(T1+T2)/2.
ISN 0060      120 CONTINUE
ISN 0061          SW1=.5
ISN 0062          TW1=.0
ISN 0063          SW2=1.-SW1
ISN 0064          TW2=1.-TW1
ISN 0065          DO 5 J=1,JJ
ISN 0066          DO 6 I=1,II
ISN 0067          TIJ=T(I,J)
ISN 0068          RO(I,J)=ROF(APO,TIJ)
ISN 0069          V(I,J)=ETAF(APO,TIJ)/ROF(APO,TIJ)
ISN 0070          AK(I,J)=POR*CLAMF(APO,TIJ)+AKM
ISN 0071          CP(I,J)=CPF(APO,TIJ)
ISN 0072        6 CONTINUE
ISN 0073        5 CONTINUE
ISN 0074          DO 15 L=1,ME
ISN 0075       15 A(L)=.0
            C
            C     MOMENTUM EQUATION
            C
ISN 0076          IF(NG.EQ.2) GO TO 400
ISN 0078          IF(NG.LE.5 .AND. NG.GE.3) GO TO 410
            C
            C     HORIZONTAL CYLINDER
            C
            C     FILL IN COEFFICIENT MATRIX A AND
            C     RIGHT HAND SIDE VECTOR SR
ISN 0080          K=-II
ISN 0081          L=-1
ISN 0082          DO 1 I=1,II
ISN 0083          L=L+1
ISN 0084          K=K+II+L
ISN 0085          H=II+1-(II-L)
ISN 0086          A(K+H)=1.
ISN 0087          SR(L+1)=.0
ISN 0088        1 CONTINUE
ISN 0089          K=K-1
ISN 0090          DO 10 J=2,JJ1
ISN 0091          L=L+1
ISN 0092          K=K+2*II+1
ISN 0093          H=II+1
ISN 0094          A(K+H)=1.
ISN 0095          SR(L+1)=.0
ISN 0096          DO 11 I=2,II1
```

```
ISN 0097          L=L+1
ISN 0098          K=K+2*II+1
ISN 0099          H=II+1
ISN 0100          A(K+H)=-(V(I,J-1)+2.*V(I,J)+V(I,J+1))/(2.*DTH2*R(I))
                 1-(R(I+1)*V(I+1,J)+2.*R(I)*V(I,J)+R(I-1)*V(I-1,J))/(2.*DR2)
ISN 0101          H=1
ISN 0102          A(K+H)=(V(I,J-1)+V(I,J))/(2.*DTH2*R(I))
ISN 0103          H=2*II+1
ISN 0104          A(K+H)=(V(I,J)+V(I,J+1))/(2.*DTH2*R(I))
ISN 0105          H=II
ISN 0106          A(K+H)=(R(I)*V(I,J)+R(I-1)*V(I-1,J))/(2.*DR2)
ISN 0107          H=II+2
ISN 0108          A(K+H)=(R(I+1)*V(I+1,J)+R(I)*V(I,J))/(2.*DR2)
ISN 0109          DROTH=(RO(I,J+1)-RO(I,J-1))/(2.*DTH)
ISN 0110          DROR=(RO(I+1,J)-RO(I-1,J))/(2.*DR)
ISN 0111          SR(L+1)=G*PER*(COS(TH(J))*DROTH+R(I)*SIN(TH(J))*DROR)
ISN 0112       11 CONTINUE
ISN 0113          L=L+1
ISN 0114          K=K+2*II+1
ISN 0115          H=II+1
ISN 0116          '(K+H)=1.
ISN 0117           (L+1)=.0
ISN 0118       1( )NTINUE
ISN 0119          DO 2 I=1,II
ISN 0120          L=L+1
ISN 0121          K=K+M+II+1-L
ISN 0122          H=II+1
ISN 0123          A(K+H)=1.
ISN 0124          SR(L+1)=.0
ISN 0125        2 CONTINUE
ISN 0126          CALL GELB(SR,A,M,N,MUD,MLD,EPS,IER)
ISN 0127          KOUNT=KOUNT+1
            C     PRINT STREAM FUNCTION
            C
ISN 0128          WRITE(6,200) KOUNT
ISN 0129          WRITE(6,218)IER
ISN 0130          WRITE(6,201) SR
            C     UPDATE STREAM FUNCTION
            C
ISN 0131          IF(KOUNT.EQ.1) GO TO 71
ISN 0133          DO 70 L= 1,M
ISN 0134       70 SLINE(L)=SW1*SLINE(L)+SW2*SR(L)
ISN 0135          GO TO 73
ISN 0136       71 DO 72 L=1,M
ISN 0137       72 SLINE(L)=SR(L)
ISN 0138       73 CONTINUE
            C     ENERGY EQUATION
ISN 0139          DO 16 L=1,ME
ISN 0140       16 A(L)=.0
            C
            C     FILL IN COEFFICIENT MATRIX A AND
            C     RIGHT HAND SIDE VECTOR TR
ISN 0141          IF(NBC.EQ.1)GOTO 42
ISN 0143          A(1)=1.
ISN 0144          TR(1)=T1
ISN 0145          GO TO 43
ISN 0146       42 CONTINUE
ISN 0147          A(1)=3.*AK(1,1)/(2.*DR)+AGAM
```

```
ISN [illegible]          [illegible]=[illegible].*AK(I,1)/DR
ISN [illegible]          [illegible]K(I,1)/(2.*DR)
ISN [illegible]          [illegible]AGAS*TGAS
ISN [illegible]       [illegible] CONTINUE
ISN [illegible]          [illegible]
ISN [illegible]          [illegible]
ISN 0154                 DO [illegible] I=2,III
ISN 0155                 L=L+1
ISN 0156                 K=K+II+1
ISN 0157                 H=II+1-(II-L)
ISN 0158                 A(K+H)=1.
ISN 0159                 [illegible]+II+1-(II-L)
ISN 0160                 A(K+H)=[illegible].
ISN 0161                 [illegible](L+1)=[illegible]
ISN 0162              [illegible] CONTINUE

ISN 0163                 L=L+1
ISN 0164                 K=K+II+1
ISN 0165                 IF(NRC.EQ.1)GOTO 45
ISN 0167                 H=II+1-(II-L)
ISN 0168                 A(K+H)=1.
ISN 0169                 TR(L+1)=T2
ISN 0170                 GOTO 46
ISN 0171              45 CONTINUE
ISN 0172                 H=II+1-(II-L)
ISN 0173                 A(K+H)=AGAC+3.*AK(II,1)/(2.*DR)
ISN 0174                 H=II-(II-L)
ISN 0175                 A(K+H)=-2.*AK(II,1)/DR
ISN 0176                 H=II-1-(II-L)
ISN 0177                 A(K+H)=AK(II,1)/(2.*DR)
ISN 0178                 TR(L+1)=AGAC*TOUT
ISN 0179              46 CONTINUE
ISN 0180                 K=K-1
ISN 0181                 DO 110 J=2,JJ1
ISN 0182                 L=L+1
ISN 0183                 K=K+2*II+1
ISN 0184                 IF(NRC.EQ.1)GOTO 52
ISN 0186                 H=II+1
ISN 0187                 A(K+H)=1.
ISN 0188                 TR(L+1)=T1
ISN 0189                 GO TO 53
ISN 0190              52 CONTINUE
ISN 0191                 H=II+1
ISN 0192                 A(K+H)=3.*AK(I,J)/(2.*DR)+AGAH
ISN 0193                 H=II+2
ISN 0194                 A(K+H)=-2.*AK(I,J)/DR
ISN 0195                 H=II+3
ISN 0196                 A(K+H)=AK(I,J)/(2.*DR)
ISN 0197                 TR(L+1)=AGAH*TGAS
ISN 0198              53 CONTINUE

ISN 0199                 DO 111 I=2,III
ISN 0200                 L=L+1
ISN 0201                 K=K+2*II+1
ISN 0202                 H=II+1
ISN 0203                 A(K+H)=(R(I+1)*AK(I+1,J)+2.*R(I)*AK(I,J)+R(I-1)*AK(I-1,J))/
                        1(2.*DR2)+(AK(I,J+1)+2.*AK(I,J)+AK(I,J-1))/(2.*R(I)*DTH2)
ISN 0204                 H=1
```

```
ISN 0205          A(K+H)=(S(I+1,J)-S(I-1,J))*CP(I,J-1)/(4.*DTH*DR)
                 1-(AK(I,J)+AK(I,J-1))/(2.*R(I)*DTH2)
ISN 0206          H=2*II+1
ISN 0207          A(K+H)=-(S(I+1,J)-S(I-1,J))*CP(I,J+1)/(4.*DTH*DR)
                 1-(AK(I,J+1)+AK(I,J))/(2.*R(I)*DTH2)
ISN 0208          H=II
ISN 0209          A(K+H)=-(S(I,J+1)-S(I,J-1))*CP(I-1,J)/(4.*DTH*DR)
                 1-(R(I)*AK(I,J)+R(I-1)*AK(I-1,J))/(2.*DR2)
ISN 0210          H=II+2
ISN 0211          A(K+H)=+(S(I,J+1)-S(I,J-1))*CP(I+1,J)/(4.*DTH*DR)
                 1-(R(I+1)*AK(I+1,J)+R(I)*AK(I,J))/(2.*DR2)
ISN 0212          TR(L+1)=.0
ISN 0213      111 CONTINUE
            C
ISN 0214          L=L+1
ISN 0215          K=K+2*II+1
ISN 0216          IF(MBC.EQ.1)GOTO 55
ISN 0218          H=II+1
ISN 0219          A(K+H)=1.
ISN 0220          TR(L+1)=T2
ISN 0221          GOTO 56
ISN 0222       55 CONTINUE
ISN 0223          H=II+1
ISN 0224          A(K+H)=AGAC+3.*AK(II,J)/(2.*DR)
ISN 0225          H=II
ISN 0226          A(K+H)=-2.*AK(II,J)/DR
ISN 0227          H=II-1
ISN 0228          A(K+H)=AK(II,J)/(2.*DR)
ISN 0229          TR(L+1)=AGAC*TOUT
ISN 0230       56 CONTINUE
ISN 0231      110 CONTINUE
            C
ISN 0232          L=L+1
ISN 0233          K=K+2*II+1
ISN 0234          IF(NBC.EQ.1)GOTO 62
ISN 0236          H=II+1
ISN 0237          A(K+H)=1.
ISN 0238          TR(L+1)=T1
ISN 0239          GO TO 63
ISN 0240       62 CONTINUE
ISN 0241          H=II+1
ISN 0242          A(K+H)=3.*AK(1,JJ)/(2.*DR)+AGAH
ISN 0243          H=II+2
ISN 0244          A(K+H)=-2.*AK(1,JJ)/DR
ISN 0245          H=II+3
ISN 0246          A(K+H)=AK(1,JJ)/(2.*DR)
ISN 0247          TR(L+1)=AGAH*TGAS
ISN 0248       63 CONTINUE
            C
ISN 0249          DO 102 I=2,III
ISN 0250          L=L+1
ISN 0251          K=K+M+II+1-L
ISN 0252          H=II+1
ISN 0253          A(K+H)=1.
ISN 0254          H=1
ISN 0255          A(K+H)=-1.
ISN 0256          TR(L+1)=.0
ISN 0257      102 CONTINUE
```

```
                  C
ISN 0258                L=L+1
ISN 0259                K=K+M+II+1-L
ISN 0260                IF(MBC.EQ.1)GOTO 65
ISN 0262                H=II+1
ISN 0263                A(K+H)=1.
ISN 0264                TR(L+1)=T2
ISN 0265                GOTO 66
ISN 0266             65 CONTINUE
ISN 0267                H=II+1
ISN 0268                A(K+H)=AGAC+3.*AK(II,JJ)/(2.*DR)
ISN 0269                H=II
ISN 0270                A(K+H)=-2.*AK(II,JJ)/DR
ISN 0271                H=II-1
ISN 0272                A(K+H)=AK(II,JJ)/(2.*DR)
ISN 0273                TR(L+1)=AGAC*TOUT
ISN 0274             66 CONTINUE
                  C     MATRIX A IS FILLED IN
                  C
ISN 0275                CALL GELB(TR,A,M,N,MUD,MLD,EPS,IER)
                  C
                  C     WRITE TEMPERATURES
                  C
ISN 0276                WRITE(6,210)
ISN 0277                WRITE(6,211) TR
                  C
                  C
                  C     CHECK THE TEMPERATURES FOR CONVERGENCE
                  C
ISN 0278                DO 1300 L=1,M
ISN 0279                IF(DABS(TR(L)-TLINE(L)).GT.EPT) GO TO 116
ISN 0281           1300 CONTINUE
ISN 0282                GO TO 115
                  C
                  C     CHECK IF KOUNT IS LESS THAN KMAX
                  C
ISN 0283            116 IF(KOUNT.EQ.KMAX) GO TO 115
                  C
                  C     UPDATE TEMPERATURES
ISN 0285                DO 80 L=1,M
ISN 0286             80 TLINE(L)=TW1*TLINE(L)+TW2*TR(L)
ISN 0287                GO TO 120
ISN 0288            115 CONTINUE
                  C
ISN 0289                T1=.0
ISN 0290                T2=.0
ISN 0291                DO 97 J=1,JJ
ISN 0292                T1=TEND(1,J)/JJ+T1
ISN 0293                T2=TEND(II,J)/JJ+T2
ISN 0294             97 CONTINUE
ISN 0295                TB=(T1+T2)/2.
                  C     CALCULATE VELOCITIES
                  C     MVR  MASS VELOCITY IN THE R DIRECTION
                  C     MVTH MASS VELOCITY IN THE TH DIRECTION
ISN 0296                DO 82 J=1,JJ1
ISN 0297                DO 81 I=1,II
ISN 0298                SEND(I,J)=S(I,J)
ISN 0299             81 MVR(I,J)=(SEND(I,J+1)-SEND(I,J))/(R(I)*DTH)
```

```
ISN 0300          82 CONTINUE
ISN 0301             DO 85 I=1,III
ISN 0302             DO 86 J=1,JJ
ISN 0303          86 MVTH(I,J)=-(SEND(I+1,J)-SEND(I,J))/DR
ISN 0304          85 CONTINUE
             C       CALCULATE THE HEAT FLUX AND K EFFECTIVE
ISN 0305             QT(1)=.0
ISN 0306             QPHT=.0
             C
ISN 0307             DO 90 J=1,JJ1
ISN 0308             T(1,J)=TEND(1,J)
ISN 0309             QPH(J)=-(-T(3,J)-T(3,J+1)+4.*(T(2,J)+T(2,J+1))-3.*(T(1,J)+
                    1T(1,J+1)))*(AK(1,J)+AK(1,J+1))/(8.*DR)*R(1)*DTH
             C
ISN 0310             CONV(1,J)=.0
ISN 0311             COND(1,J)=(T(1,J)+T(1,J+1)-T(2,J)-T(2,J+1))*(AK(1,J)+AK(1,J+1))
                    1/(4.*DR)*R(1)*DTH
             C
ISN 0312             Q(1,J)=CONV(1,J)+COND(1,J)
ISN 0313             QPHT=QPHT+QPH(J)
ISN 0314          90 QT(1)=QT(1)+Q(1,J)
             C
ISN 0315             DO 91 I=2,III
ISN 0316             QT(I)=.0
ISN 0317             DO 92 J=1,JJ1
ISN 0318             T(I,J)=TEND(I,J)
ISN 0319             COND(I,J)=(T(I-1,J)+T(I-1,J+1)-T(I+1,J)-T(I+1,J+1))
                    1*(AK(I,J)+AK(I,J+1))/(8.*DR)*R(I)*DTH
ISN 0320             CONV(I,J)= MVR(I,J)*(T(I,J)+T(I,J+1))*(CP(I,J)+CP(I,J+1))/4.
                    1*R(I)*DTH
ISN 0321             Q(I,J)=CONV(I,J)+COND(I,J)
ISN 0322          92 QT(I)=QT(I)+Q(I,J)
ISN 0323          91 CONTINUE
             C
ISN 0324             QT(II)=.0
ISN 0325             QPCT=.0
ISN 0326             DO 93 J=1,JJ1
ISN 0327             T(I,J)=TEND(I,J)
ISN 0328             QPC(J)=(-T(II-2,J)-T(II-2,J+1)+4.*(T(II-1,J)+T(II-1,J+1))
                    1-3.*(T(II,J)+T(II,J+1)))*(AK(II,J)+AK(II,J+1))/(8.*DR)*R(II)*DTH
             C
ISN 0329             CONV(II,J)=.0
ISN 0330             COND(II,J)=(T(II-1,J)+T(II-1,J+1)-T(II,J)-T(II,J+1))
                    1*(AK(II,J)+AK(II,J+1))/(4.*DR)*R(II)*DTH
ISN 0331             Q(II,J)=CONV(II,J)+COND(II,J)
ISN 0332             QPCT=QPCT+QPC(J)
ISN 0333          93 QT(II)=QT(II)+Q(II,J)
             C
ISN 0334             QTAV=.0
ISN 0335             DO 96 I=1,II
ISN 0336             QTAV=QTAV+QT(I)/II
ISN 0337          96 CONTINUE
ISN 0338             AKAV=QTAV*ALOG(R2/R1)/((T1-T2)*3.1416)
ISN 0339             AKEFC=QPCT*ALOG(R2/R1)/((T1-T2)*3.1416)
ISN 0340             AKEFH=QPHT*ALOG(R2/R1)/((T1-T2)*3.1416)
ISN 0341             GO TO 450
ISN 0342         400 CONTINUE
             C
```

```
              C       VERTICAL CYLINDER
              C
              C       FILL IN COEFFICIENT MATRIX A AND
              C       RIGHT HAND SIDE VECTOR SR
ISN 0343              K=-II
ISN 0344              L=1
ISN 0345              DO 601 I=1,II
ISN 0346              L=L+1
ISN 0347              K=K+II+L
ISN 0348              H=II+1-(II-L)
ISN 0349              A(K+H)=1.
ISN 0350              SR(L+1)=.0
ISN 0351          601 CONTINUE
ISN 0352              K=K-1
ISN 0353              DO 610 J=2,JJ1
ISN 0354              L=L+1
ISN 0355              K=K+2*II+1
ISN 0356              IF(NPE.EQ.1) GO TO 612
ISN 0358              H=II+1
ISN 0359              A(K+H)=1.
ISN 0360              SR(L+1)=.0
ISN 0361              GO TO 613
ISN 0362          612 CONTINUE
ISN 0363              H=1
ISN 0364              A(K+H)=C1*(V(1,J)+V(1,J-1))/(V(1,J))
ISN 0365              H=2*II+1
ISN 0366              A(K+H)=C1*(V(1,J)+V(1,J+1))/(V(1,J))
ISN 0367              H=II+1
ISN 0368              A(K+H)=-1./DR-C1*(V(1,J-1)+2.*V(1,J)+V(1,J+1))/(V(1,J))
ISN 0369              H=II+2
ISN 0370              A(K+H)=1./DR
ISN 0371              SR(L+1)=R(1)*PER/V(1,J)*(G*RO(1,J)+DPDX)
ISN 0372          613 CONTINUE
ISN 0373              DO 611 I=2,II1
ISN 0374              L=L+1
ISN 0375              K=K+2*II+1
ISN 0376              H=II+1
ISN 0377              A(K+H)=-(V(I,J-1)+2.*V(I,J)+V(I,J+1))/(2.*DTH2*R(I))
                     1-(V(I+1,J)/R(I+1)+2.*V(I,J)/R(I)+V(I-1,J)/R(I-1))/(2.*DR2)
ISN 0378              H=1
ISN 0379              A(K+H)=(V(I,J-1)+V(I,J))/(2.*DTH2*R(I))
ISN 0380              H=2*II+1
ISN 0381              A(K+H)=(V(I,J)+V(I,J+1))/(2.*DTH2*R(I))
ISN 0382              H=II
ISN 0383              A(K+H)=(V(I,J)/R(I)+V(I-1,J)/R(I-1))/(2.*DR2)
ISN 0384              H=II+2
ISN 0385              A(K+H)=(V(I+1,J)/R(I+1)+V(I,J)/R(I))/(2.*DR2)
ISN 0386              DROTH=(RO(I,J+1)-RO(I,J-1))/(2.*DTH)
ISN 0387              DROR=(RO(I+1,J)-RO(I-1,J))/(2.*DR)
ISN 0388              SR(L+1)=G*PER*DROR
ISN 0389          611 CONTINUE
ISN 0390              L=L+1
ISN 0391              K=K+2*II+1
ISN 0392              H=II+1
ISN 0393              A(K+H)=1.
ISN 0394              SR(L+1)=.0
ISN 0395          610 CONTINUE
ISN 0396              DO 602 I=1,II
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ISN 0397          L=L+1
ISN 0398          K=K+M+II+1-L
ISN 0399          H=II+1
ISN 0400          A(K+H)=1.
ISN 0401          SR(L+1)=.0
ISN 0402      602 CONTINUE
ISN 0403          CALL GELB(SR,A,M,N,MUD,MLO,EPS,IER)
ISN 0404          KOUNT=KOUNT+1
            C     PRINT STREAM FUNCTION
            C
ISN 0405          WRITE(6,200) KOUNT
ISN 0406          WRITE(6,218)IER
ISN 0407          WRITE(6,201) SR
            C     UPDATE STREAM FUNCTION
            C
ISN 0408          IF(KOUNT.EQ.1) GO TO 671
ISN 0410          DO 670 L=1,M
ISN 0411      670 SLINE(L)=SW1*SLINE(L)+SW2*SR(L)
ISN 0412          GO TO 673
ISN 0413      671 DO 672 L=1,M
ISN 0414      672 SLINE(L)=SR(L)
ISN 0415      673 CONTINUE
            C     ENERGY EQUATION
ISN 0416          DO 616 L=1,ME
ISN 0417      616 A(L)=.0
            C
            C     FILL IN COEFFICIENT MATRIX A AND
            C     RIGHT HAND SIDE VECTOR TR
ISN 0418          IF(NBC.EQ.1) GO TO 642
ISN 0420          A(1)=1.
ISN 0421          TR(1)=T1
ISN 0422          GO TO 643
ISN 0423      642 CONTINUE
ISN 0424          A(1)=3.*AK(1,1)/(2.*DR)+AGAH
ISN 0425          A(2)=-2.*AK(1,1)/DR
ISN 0426          A(3)=AK(1,1)/(2.*DR)
ISN 0427        . TR(1)=AGAH*TGAS
ISN 0428      643 CONTINUE
ISN 0429          L=0
ISN 0430          K=0
ISN 0431          DO 6101 I=2,III
ISN 0432          L=L+1
ISN 0433          K=K+II+L
ISN 0434          H=II+1-(II-L)
ISN 0435          A(K+H)=1.
ISN 0436          H=2*II+1-(II-L)
ISN 0437          A(K+H)=-1.
ISN 0438          TR(L+1)=.0
ISN 0439     6101 CONTINUE
            C
ISN 0440          L=L+1
ISN 0441          K=K+II+L
ISN 0442          IF(MBC.EQ.1) GO TO 645
ISN 0444          H=II+1-(II-L)
ISN 0445          A(K+H)=1.
ISN 0446          TR(L+1)=T2
ISN 0447          GO TO 646
ISN 0448      645 CONTINUE
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```
ISN 0449          H=II+1-(II-L)
ISN 0450          A(K+H)=AGAC+3.*AK(II,1)/(2.*DR)
ISN 0451          H=II-(II-L)
ISN 0452          A(K+H)=-2.*AK(II,1)/DR
ISN 0453          H=II-1-(II-L)
ISN 0454          A(K+H)=AK(II,1)/(2.*DR)
ISN 0455          TR(L+1)=AGAC*TOUT
ISN 0456      646 CONTINUE
ISN 0457          K=K-1
ISN 0458          DO 6110 J=2,JJ1
ISN 0459          L=L+1
ISN 0460          K=K+2*II+1
ISN 0461          IF(NBC.EQ.1) GO TO 652
ISN 0463          H=II+1
ISN 0464          A(K+H)=1.
ISN 0465          TR(L+1)=T1
                C
ISN 0466          GO TO 653
ISN 0467      652 CONTINUE
ISN 0468          H=II+1
ISN 0469          A(K+H)=3.*AK(1,J)/(2.*DR)+AGAH
ISN 0470          H=II+2
ISN 0471          A(K+H)=-2.*AK(1,J)/DR
ISN 0472          H=II+3
ISN 0473          A(K+H)=AK(1,J)/(2.*DR)
ISN 0474          TR(L+1)=AGAH*TGAS
ISN 0475      653 CONTINUE
ISN 0476          DO 6111 I=2,III
ISN 0477          L=L+1
ISN 0478          K=K+2*II+1
ISN 0479          H=II+1
ISN 0480          A(K+H)=(R(I+1)*AK(I+1,J)+2.*R(I)*AK(I,J)+R(I-1)*AK(I-1,J))/
                 1(2.*DR2)+(AK(I,J+1)+2.*AK(I,J)+AK(I,J-1))*R(I)/(2.*DTH2)
ISN 0481          H=1
ISN 0482          A(K+H)=(S(I+1,J)-S(I-1,J))*CP(I,J-1)/(4.*DTH*DR)
                 1 -(AK(I,J)+AK(I,J-1))*R(I)/(2.*DTH2)
ISN 0483          H=2*II+1
ISN 0484          A(K+H)=-(S(I+1,J)-S(I-1,J))*CP(I,J+1)/(4.*DTH*DR)
                 1 -(AK(I,J+1)+AK(I,J))*R(I)/(2.*DTH2)
ISN 0485          H=II
ISN 0486          A(K+H)=-(S(I,J+1)-S(I,J-1))*CP(I-1,J)/(4.*DTH*DR)
                 1-(R(I)*AK(I,J)+R(I-1)*AK(I-1,J))/(2.*DR2)
ISN 0487          H=II+2
ISN 0488          A(K+H)=+(S(I,J+1)-S(I,J-1))*CP(I+1,J)/(4.*DTH*DR)
                 1-(R(I+1)*AK(I+1,J)+R(I)*AK(I,J))/(2.*DR2)
ISN 0489          TR(L+1)=.0
ISN 0490     6111 CONTINUE
                C
ISN 0491          L=L+1
ISN 0492          K=K+2*II+1
ISN 0493          IF(MBC.EQ.1) GO TO 655
ISN 0495          H=II+1
ISN 0496          A(K+H)=1.
ISN 0497          TR(L+1)=T2
ISN 0498          GO TO 656
ISN 0499      655 CONTINUE
ISN 0500          H=II+1
ISN 0501          A(K+H)=AGAC+3.*AK(II,J)/(2.*DR)
```

```
ISN 0502          H=II
ISN 0503          A(K+H)=-2.*AK(II,J)/DR
ISN 0504          H=II-1
ISN 0505          A(K+H)=AK(II,J)/(2.*DR)
ISN 0506          TR(L+1)=AGAC*TOUT
ISN 0507      656 CONTINUE
ISN 0508     6110 CONTINUE
            C
ISN 0509          L=L+1
ISN 0510          K=K+2*II+1
ISN 0511          IF(NBC.EQ.1) GO TO 662
ISN 0513          H=II+1
ISN 0514          A(K+H)=1.
ISN 0515          TR(L+1)=T1
ISN 0516          GO TO 663
ISN 0517      662 CONTINUE
ISN 0518          H=II+1
ISN 0519          A(K+H)=3.*AK(1,JJ)/(2.*DR)+AGAH
ISN 0520          H=II+2
ISN 0521          A(K+H)=-2.*AK(1,JJ)/DR
ISN 0522          H=II+3
ISN 0523          A(K+H)=AK(1,JJ)/(2.*DR)
ISN 0524          TR(L+1)=AGAH*TGAS
ISN 0525      663 CONTINUE
            C
ISN 0526          DO 6102 I=2,III
ISN 0527          L=L+1
ISN 0528          K=K+M+II+1-L
ISN 0529          H=II+1
ISN 0530          A(K+H)=1.
ISN 0531          H=1
ISN 0532          A(K+H)=-1.
ISN 0533          TR(L+1)=.0
ISN 0534     6102 CONTINUE
            C
ISN 0535          L=L+1
ISN 0536          K=K+M+II+1-L
ISN 0537          IF(MBC.EQ.1) GO TO 665
ISN 0539          H=II+1
ISN 0540          A(K+H)=1.
ISN 0541          TR(L+1)=T2
ISN 0542          GO TO 666
ISN 0543      665 CONTINUE
ISN 0544          H=II+1
ISN 0545          A(K+H)=AGAC+3.*AK(II,JJ)/(2.*DR)
ISN 0546          H=II
ISN 0547          A(K+H)=-2.*AK(II,JJ)/DR
ISN 0548          H=II-1
ISN 0549          A(K+H)=AK(II,JJ)/(2.*DR)
ISN 0550          TR(L+1)=AGAC*TOUT
ISN 0551      666 CONTINUE
            C     MATRIX A IS FILLED IN
            C
ISN 0552          CALL GELB(TR,A,M,N,MUO,MLD,EPS,IER)
            C
            C     WRITE TEMPERATURES
            C
ISN 0553          WRITE(6,210)
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ISN 0554            WRITE(6,211) TR
              C
              C
              C      CHECK THE TEMPERATURES FOR CONVERGENCE
              C
ISN 0555            DO 6300 L=1,M
ISN 0556            IF(DABS(TR(L)-TLINE(L)).GT.EPT) GO TO6116
ISN 0558       6300 CONTINUE
ISN 0559            GO TO 6115
              C
              C      CHECK IF KOUNT IS LESS THAN KMAX
              C
ISN 0560       6116 IF(KOUNT.EQ.KMAX) GO TO 6115
              C
              C      UPDATE TEMPERATURES
ISN 0562            DO 690 L=1,M
ISN 0563        690 TLINE(L)=TW1*TLINE(L)+TW2*TR(L)
ISN 0564            GO TO 120
ISN 0565       6115 CONTINUE
              C
ISN 0566            T1=.0
ISN 0567            T2=.0
ISN 0568            DO 697 J=1,JJ
ISN 0569            T1=TEND(1,J)/JJ+T1
ISN 0570            T2=TEND(II,J)/JJ+T2
ISN 0571        697 CONTINUE
ISN 0572            TR=(T1+T2)/2.
              C      CALCULATE VELOCITIES
              C      MVR  MASS VELOCITY IN THE R DIRECTION
              C      MVTH MASS VELOCITY IN THE TH DIRECTION
ISN 0573            DO 682 J=1,JJ1
ISN 0574            DO 681 I=1,II
ISN 0575            SEND(I,J)=S(I,J)
ISN 0576        681 MVR(I,J)=(S(I,J+1)-S(I,J))/(R(I)*DTH)
ISN 0577        682 CONTINUE
ISN 0578            DO 685 I=1,II1
ISN 0579            DO 686 J=1,JJ
ISN 0580        686 MVTH(I,J)=-(S(I+1,J)-S(I,J))/(.5*DR*(R(I)+R(I+1)))
ISN 0581        685 CONTINUE
              C
              C      CALCULATE THE HEAT FLUX AND K EFFECTIVE
ISN 0582            QT(1)=.0
ISN 0583            QPHT=.0
              C
ISN 0584            DO 670 J=1,JJ1
ISN 0585            T(I,J)=TEND(I,J)
              C
ISN 0586            COND(1,J)=(T(1,J)+T(1,J+1)-T(2,J)-T(2,J+1))*(AK(1,J)+AK(1,J+1))
                   1/(4.*DR)*DTH*2.*3.1416*R(1)
ISN 0587            CONV(1,J)=MVR(1,J)*(T(1,J)+T(1,J+1))*(CP(1,J)+CP(1,J+1))/4.
                   1*DTH*2.*3.1416*R(1)
ISN 0588            IF(MVR(1,J).LT. .0) CONV(1,J)=MVR(1,J)*(T(1,J)+T(2,J)
                   1+T(1,J+1)+T(2,J+1))*(CP(1,J)+CP(1,J+1))/8.*DTH*2.*3.1416*R(1)
ISN 0590            QPH(J)=-(-T(3,J)-T(3,J+1)+4.*(T(2,J)+T(2,J+1))-3.*(T(1,J)+T(1,J+1)
                   1))*(AK(1,J)+AK(1,J+1))/(8.*DR)*DTH*2.*3.1416*R(1)
ISN 0591            QPH(J)=QPH(J)+CONV(1,J)
              C
ISN 0592            Q(1,J)=CONV(1,J)+COND(1,J)
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ISN 0593          QPHT=QPHT+QPH(J)
ISN 0594      690 QT(I)=QT(I)+Q(I,J)
              C
ISN 0595          DO 691 I=2,III
ISN 0596          QT(I)=.0
ISN 0597          DO 692 J=1,JJ1
ISN 0598          T(I,J)=TEND(I,J)
ISN 0599          COND(I,J)=(T(I-1,J)+T(I-1,J+1)-T(I+1,J)-T(I+1,J+1))
                 1*(AK(I,J)+AK(I,J+1))/(8.*DR)*DTH*2.*3.1416*R(I)
ISN 0600          CONV(I,J)=MVR(I,J)*(T(I,J)+T(I,J+1))*(CP(I,J)+CP(I,J+1))/4.
                 1*DTH*2.*3.1416*R(I)
ISN 0601          Q(I,J)=CONV(I,J)+COND(I,J)
ISN 0602      692 QT(I)=QT(I)+Q(I,J)
ISN 0603      691 CONTINUE
ISN 0604          DO 693 J=1,JJ1
ISN 0605          T(I,J)=TEND(I,J)
ISN 0606          QPC(J)=(-T(II-2,J)-T(II-2,J+1)+4.*(T(II-1,J)+T(II-1,J+1))-3.*(T(II
                 1 ,J)+T(II,J+1)))*(AK(II,J)+AK(II,J+1))/(8.*DR)*DTH*2.*3.1416*R(II)
              C
ISN 0607          CONV(II,J)=.0
ISN 0608          COND(II,J)=(T(II-1,J)+T(II-1,J+1)-T(II,J)-T(II,J+1))
                 1*(AK(II,J)+AK(II,J+1))/(4.*DR)*DTH*2.*3.1416*R(II)
ISN 0609          Q(II,J)=CONV(II,J)+COND(II,J)
              C
ISN 0610          QPCT=QPCT+QPC(J)
ISN 0611      693 QT(II)=QT(II)+Q(II,J)
ISN 0612          DO 696 I=1,II
ISN 0613          QTAV=QTAV+QT(I)/II
ISN 0614      696 CONTINUE
ISN 0615          AKAV=QTAV*ALOG(R2/R1)/((T1-T2)*3.1416*2.*AL)
ISN 0616          AKEFH=QPHT*ALOG(R2/R1)/((T1-T2)*3.1416*2.*AL)
ISN 0617          AKEFC=QPCT*ALOG(R2/R1)/((T1-T2)*3.1416*2.*AL)
              C
ISN 0618          GO TO 450
ISN 0619      410 CONTINUE
              C
              C       RECTANGULAR GEOMETRIES
              C
              C       FILL IN COEFFICIENT MATRIX A AND
              C       RIGHT HAND SIDE VECTOR SR
              C
ISN 0620          K=-II
ISN 0621          L=-1
ISN 0622          DO 701 I=1,II
ISN 0623          L=L+1
ISN 0624          K=K+II+L
ISN 0625          M=II+1-(II-L)
ISN 0626          A(K+M)=1.
ISN 0627          SR(L+1)=.0
ISN 0628      701 CONTINUE
ISN 0629          K=K-1
ISN 0630          DO 710 J=2,JJ1
ISN 0631          L=L+1
ISN 0632          K=K+2*II+1
ISN 0633          IF(NPF.EQ.1) GO TO 712
ISN 0635          M=II+1
ISN 0636          A(K+M)=1.
ISN 0637          SR(L+1)=.0
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ISN 0638          GO TO 713
ISN 0639      712 CONTINUE
ISN 0640          H=1
ISN 0641          A(K+H)=C1*(V(1,J)+V(1,J-1))/(V(1,J))
ISN 0642          H=2*II+1
ISN 0643          A(K+H)=C1*(V(1,J)+V(1,J+1))/(V(1,J))
ISN 0644          H=II+1
ISN 0645          A(K+H)=-1./DR-C1*(V(1,J-1)+2.*V(1,J)+V(1,J+1))/(V(1,J))
ISN 0646          H=II+2
ISN 0647          A(K+H)=1./DR
ISN 0648          SR(L+1)=PER/V(1,J)*(-GTH*RO(1,J)+DPDX+TCP*GR)
ISN 0649      713 CONTINUE
ISN 0650          DO 711 I=2,III
ISN 0651          L=L+1
ISN 0652          K=K+2*II+1
ISN 0653          H=II+1
ISN 0654          A(K+H)=-(V(I,J-1)+2.*V(I,J)+V(I,J+1))/(2.*DTH2)
                 1-(V(I+1,J)+2.*V(I,J)+V(I-1,J))/(2.*DR2)
ISN 0655          H=1
ISN 0656          A(K+H)=(V(I,J-1)+V(I,J))/(2.*DTH2)
ISN 0657          H=2*II+1
ISN 0658          A(K+H)=(V(I,J)+V(I,J+1))/(2.*DTH2)
ISN 0659          H=II
ISN 0660          A(K+H)=(V(I,J)+V(I-1,J))/(2.*DR2)
ISN 0661          H=II+2
ISN 0662          A(K+H)=(V(I+1,J)+V(I,J))/(2.*DR2)
ISN 0663          DROTH=(RO(I,J+1)-RO(I,J-1))/(2.*DTH)
ISN 0664          DROR=(RO(I+1,J)-RO(I-1,J))/(2.*DR)
ISN 0665          SR(L+1)=PER*(GR*DROTH-GTH*DROR)
ISN 0666      711 CONTINUE
ISN 0667          L=L+1
ISN 0668          K=K+2*II+1
ISN 0669          H=II+1
ISN 0670          A(K+H)=1.
ISN 0671          SR(L+1)=.0
ISN 0672      710 CONTINUE
ISN 0673          DO 702 I=1,II
ISN 0674          L=L+1
ISN 0675          K=K+M+II+1-L
ISN 0676          H=II+1
ISN 0677          A(K+H)=1.
ISN 0678          SR(L+1)=.0
ISN 0679      702 CONTINUE
ISN 0680          CALL GELB(SR,A,M,N,MUD,MLO,EPS,IER)
ISN 0681          KOUNT=KOUNT+1
              C   PRINT STREAM FUNCTION
              C
ISN 0682          WRITE(6,200) KOUNT
ISN 0683          WRITE(6,218)IER
ISN 0684          WRITE(6,201) SR
              C   CALCULATE VELOCITIES
              C   UPDATE STREAM FUNCTION
              C
ISN 0685          IF(KOUNT.EQ.1) GO TO 771
ISN 0687          DO 770 L=1,M
ISN 0688      770 SLINE(L)=SW1*SLINE(L)+SW2*SR(L)
ISN 0689          GO TO 773
ISN 0690      771 DO 772 L=1,M
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ISN 0691        772 SETMP(L)=SR(L)
ISN 0692        773 CONTINUE
             C      ENERGY EQUATION
ISN 0693            DO 716 I=1,ME
ISN 0694        716 A(I)=.0
             C
             C      FILL IN COEFFICIENT MATRIX A AND
             C      RIGHT HAND SIDE VECTOR TR
             C
ISN 0695            IF(NBC.EQ.1) GO TO 742
ISN 0697            A(1)=1.
ISN 0698            TR(1)=T1
ISN 0699            GO TO 743
ISN 0700        742 CONTINUE
ISN 0701            A(1)=3.*AK(1,1)/(2.*DR)+AGAH
ISN 0702            A(2)=-2.*AK(1,1)/DR
ISN 0703            A(3)=AK(1,1)/(2.*DR)
ISN 0704            TR(1)=AGAH*TGAS
ISN 0705        743 CONTINUE
ISN 0706            L=0
ISN 0707            K=0
ISN 0708            DO 7101 I=2,II1
ISN 0709            L=L+1
ISN 0710            K=K+II+L
ISN 0711            H=II+1-(II-L)
ISN 0712            A(K+H)=1.
ISN 0713            H=2*II+1-(II-L)
ISN 0714            A(K+H)=-1.
ISN 0715            TR(L+1)=.0
ISN 0716       7101 CONTINUE
             C
ISN 0717            L=L+1
ISN 0718            K=K+II+L
ISN 0719            IF(MBC.EQ.1) GO TO 745
ISN 0721            H=II+1-(II-L)
ISN 0722            A(K+H)=1.
ISN 0723            TR(L+1)=T2
ISN 0724            GO TO 746
ISN 0725        745 CONTINUE
ISN 0726            H=II+1-(II-L)
ISN 0727            A(K+H)=AGAC+3.*AK(II,1)/(2.*DR)
ISN 0728            H=II-(II-L)
ISN 0729            A(K+H)=-2.*AK(II,1)/DR
ISN 0730            H=II-1-(II-L)
ISN 0731            A(K+H)=AK(II,1)/(2.*DR)
ISN 0732            TR(L+1)=AGAC*TOUT
ISN 0733        746 CONTINUE
ISN 0734            K=K-1
ISN 0735            DO 7110 J=2,JJ1
ISN 0736            L=L+1
ISN 0737            K=K+2*II+1
ISN 0738            IF(NBC.EQ.1) GO TO 752
ISN 0740            H=II+1
ISN 0741            A(K+H)=1.
ISN 0742            TR(L+1)=T1
ISN 0743            GO TO 753
ISN 0744        752 CONTINUE
ISN 0745            H=II+1
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ISN 0746          A(K+H)=3.*AK(1,J)/(2.*DR)+AGAH
ISN 0747          H=II+2
ISN 0748          A(K+H)=-2.*AK(1,J)/DR
ISN 0749          H=II+3
ISN 0750          A(K+H)=AK(1,J)/(2.*DR)
ISN 0751          TR(L+1)=AGAH*TGAS
ISN 0752      753 CONTINUE
              C
ISN 0753          DO 7111 I=2,III
ISN 0754          L=L+1
ISN 0755          K=K+2*II+1
ISN 0756          H=II+1
ISN 0757          A(K+H)=(AK(I+1,J)+2.*AK(I,J)+AK(I-1,J))/(2.*DR2)
                 1+(AK(I,J+1)+2.*AK(I,J)+AK(I,J-1))/(2.*DTH2)
ISN 0758          H=1
ISN 0759          A(K+H)=(S(I+1,J)-S(I-1,J))*CP(I,J-1)/(4.*DTH*DR)
                 1-(AK(I,J-1)+AK(I,J))/(2.*DTH2)
ISN 0760          H=2*II+1
ISN 0761          A(K+H)=-(S(I+1,J)-S(I-1,J))*CP(I,J+1)/(4.*DTH*DR)
                 1-(AK(I,J+1)+AK(I,J))/(2.*DTH2)
ISN 0762          H=II
ISN 0763          A(K+H)=-(S(I,J+1)-S(I,J-1))*CP(I-1,J)/(4.*DTH*DR)
                 1-(AK(I-1,J)+AK(I,J))/(2.*DR2)
ISN 0764          H=II+2
ISN 0765          A(K+H)=(S(I,J+1)-S(I,J-1))*CP(I+1,J)/(4.*DTH*DR)
                 1-(AK(I+1,J)+AK(I,J))/(2.*DR2)
ISN 0766          TR(L+1)=.0
ISN 0767     7111 CONTINUE
              C
ISN 0768          L=L+1
ISN 0769          K=K+2*II+1
ISN 0770          IF(MBC.EQ.1) GO TO 755
ISN 0772          H=II+1
ISN 0773          A(K+H)=1.
ISN 0774          TR(L+1)=T2
ISN 0775          GO TO 756
ISN 0776      755 CONTINUE
ISN 0777          H=II+1
ISN 0778          A(K+H)=AGAC+3.*AK(II,J)/(2.*DR)
ISN 0779          H=II
ISN 0780          A(K+H)=-2.*AK(II,J)/DR
ISN 0781          H=II-1
ISN 0782          A(K+H)=AK(II,J)/(2.*DR)
ISN 0783          TR(L+1)=AGAC*TOUT
ISN 0784      756 CONTINUE
ISN 0785     7110 CONTINUE
              C
ISN 0786          L=L+1
ISN 0787          K=K+2*II+1
ISN 0788          IF(NBC.EQ.1) GO TO 762
ISN 0790          H=II+1
ISN 0791          A(K+H)=1.
ISN 0792          TR(L+1)=T1
ISN 0793          GO TO 763
ISN 0794      762 CONTINUE
ISN 0795          H=II+1
ISN 0796          A(K+H)=3.*AK(1,JJ)/(2.*DR)+AGAH
ISN 0797          H=II+2
```

```
ISN 0798              A(K+H)=-2.*AK(1,JJ)/DR
ISN 0799              H=II+1
ISN 0800              A(K+H)=AK(1,JJ)/(2.*DR)
ISN 0801              TR(L+1)=AGAH*TGAS
ISN 0802         763  CONTINUF
               C
ISN 0803              DO 7102 I=2,III
ISN 0804              L=I+1
ISN 0805              K=K+M+II+1-L
ISN 0806              H=II+1
ISN 0807              A(K+H)=1.
ISN 0808              H=1
ISN 0809              A(K+H)=-1.
ISN 0810              TR(L+1)=.0
ISN 0811        7102  CONTINUE
               C
ISN 0812              L=L+1
ISN 0813              K=K+M+II+1-L
ISN 0814              IF(MBC.EQ.1) GO TO 765
ISN 0816              H=II+1
ISN 0817              A(K+H)=1.
ISN 0818              TR(L+1)=T2
ISN 0819              GO TO 766
ISN 0820         765  CONTINUE
ISN 0821              H=II+1
ISN 0822              A(K+H)=AGAC+3.*AK(II,JJ)/(2.*DR)
ISN 0823              H=II
ISN 0824              A(K+H)=-2.*AK(II,JJ)/DR
ISN 0825              H=II-1
ISN 0826              A(K+H)=AK(II,JJ)/(2.*DR)
ISN 0827              TR(L+1)=AGAC*TOUT
ISN 0828         766  CONTINUE
               C      MATRIX A IS FILLED IN
               C
ISN 0829              CALL GELB(TR,A,M,N,MUD,MLD,EPS,IER)
               C
               C      WRITE TEMPERATURES
               C
ISN 0830              WRITE(6,210)
ISN 0831              WRITE(6,211) TR
               C
               C
               C      CHECK THE TEMPERATURES FOR CONVERGENCE
               C
ISN 0832              DO 7300 L=1,M
ISN 0833              IF(DABS(TR(L)-TLINE(L)).GT.EPT) GO TO7116
ISN 0835        7300  CONTINUE
ISN 0836              GO TO 7115
               C
               C      CHECK IF KOUNT IS LESS THAN KMAX
               C
ISN 0837        7116  IF(KOUNT.EQ.KMAX) GO TO 7115
               C
               C      UPDATE TEMPERATURES
ISN 0839              DO 780 L=1,M
ISN 0840         780  TLINE(L)=TW1*TLINE(L)+TW2*TR(L)
ISN 0841              GO TO 120
ISN 0842        7115  CONTINUE
```

```
                C
ISN 0843               T1=.0
ISN 0844               T2=.0
ISN 0845               DO 797 J=1,JJ
ISN 0846               T1=TEND(1,J)/JJ+T1
ISN 0847               T2=TEND(II,J)/JJ+T
ISN 0848           797 CONTINUE
ISN 0849               TH=(T1+T2)/2.
                C      MVR  MASS VELOCITY IN THE R DIRECTION
                C      MVTH MASS VELOCITY IN THE TH DIRECTION
ISN 0850               DO 782 J=1,JJ1
ISN 0851               DO 781 I=1,II
ISN 0852               SEND(I,J)=S(I,J)
ISN 0853           781 MVR(I,J)=(SEND(I,J+1)-SEND(I,J))/DTH
ISN 0854           782 CONTINUE
ISN 0855               DO 785 I=1,III
ISN 0856               DO 786 J=1,JJ
ISN 0857           786 MVTH(I,J)=-(SEND(I+1,J)-SEND(I,J))/DR
ISN 0858           785 CONTINUE
                C      CALCULATE THE HEAT FLUX-AND K EFFECTIVE
ISN 0859               QT(1)=.0
ISN 0860               QPHT=.0
                C
ISN 0861               DO 790 J=1,JJ1
ISN 0862               T(1,J)=TEND(1,J)
ISN 0863               QPH(J)=-(-T(3,J)-T(3,J+1)+4.*(T(2,J)+T(2,J+1))-3.*(T(1,J)+
                      1T(1,J+1)))*(AK(1,J)+AK(1,J+1))/(8.*DR)*DTH
                C
ISN 0864               CONV(1,J)=MVR(1,J)*(T(1,J)+T(1,J+1))*(CP(1,J)+CP(1,J+1))/4.*DTH
ISN 0865               IF(MVR(1,J).LT. .0) CONV(1,J)=MVR(1,J)*(T(1,J)+T(2,J)
                      1+T(1,J+1)+T(2,J+1))*(CP(1,J)+CP(1,J+1))/8.*DTH
ISN 0867               QPH(J)=QPH(J)+CONV(1,J)
ISN 0868               COND(1,J)=(T(1,J)+T(1,J+1)-T(2,J)-T(2,J+1))*(AK(1,J)+AK(1,J+1))
                      1/(4.*DR)*DTH
                C
ISN 0869               Q(1,J)=CONV(1,J)+COND(1,J)
ISN 0870               QPHT=QPHT+QPH(J)
ISN 0871           790 QT(1)=QT(1)+Q(1,J)
                C
ISN 0872               DO 791 I=2,III
ISN 0873               QT(I)=.0
ISN 0874               DO 792 J=1,JJ1
ISN 0875               T(I,J)=TEND(I,J)
ISN 0876               COND(I,J)=(T(I-1,J)+T(I-1,J+1)-T(I+1,J)-T(I+1,J+1))
                      1*(AK(I,J)+AK(I,J+1))/(8.*DR)*DTH
ISN 0877               CONV(I,J)= MVR(I,J)*(T(I,J)+T(I,J+1))*(CP(I,J)+CP(I,J+1))/4.*DTH
ISN 0878               Q(I,J)=CONV(I,J)+COND(I,J)
ISN 0879           792 QT(I)=QT(I)+Q(I,J)
ISN 0880           791 CONTINUE
                C
ISN 0881               QT(II)=.0
ISN 0882               QPCT=.0
ISN 0883               DO 793 J=1,JJ1
ISN 0884               T(I,J)=TEND(I,J)
ISN 0885               QPC(J)=(-T(II-2,J)-T(II-2,J+1)+4.*(T(II-1,J)+T(II-1,J+1))
                      1-3.*(T(II,J)+T(II,J+1)))*(AK(II,J)+AK(II,J+1))/(8.*DR)*DTH
                C
ISN 0886               CONV(II,J)=.0
```

```
ISN 0887             COND(II,J)=(T(II-1,J)+T(II-1,J+1)-T(II,J)-T(II,J+1))
                    1*(AK(II,J)+AK(II,J+1))/(4.*DR)*DTH
ISN 0888             Q(II,J)=CONV(II,J)+COND(II,J)
ISN 0889             QPCT=QPCT+QPC(J)
ISN 0890        793  QT(II)=QT(II)+Q(II,J)
              C
ISN 0891             QTAV=.0
ISN 0892             DO 796 I=1,II
ISN 0893             QTAV=QTAV+QT(I)/II
ISN 0894        796  CONTINUE
ISN 0895             AKAV=QTAV/((T1-T2)*AL)*(R2-R1)
ISN 0896             AKEFC=QPCT/((T1-T2)*AL)      *(R2-R1)
ISN 0897             AKEFH=QPHT/((T1-T2)*AL)      *(R2-R1)
ISN 0898        450  CONTINUE
              C      CALCULATE RALEIGH NUMBER RA
              C
ISN 0899             ANU=ETAF(APO,TB)/(ROF(APO,TB)
ISN 0900             ANUO=ETAF(APO,T1)/ROF(APO,T1)
ISN 0901             AKO=PIR*CLAMF(APO,TB)+AKM
ISN 0902             ALF=AKO/(ROF(APO,TB)*CPF(APO,TB) )
              C
ISN 0903             TF=273.16/TB
ISN 0904             DROT=-7.188E-3*APO*TF*TF*(1.+.0804*APO*TF**5)
ISN 0905             BETA=-DROT/ROF(APO,TB)
ISN 0906             RA =G*PER*(T1-T2)*(R2-R1)*BETA/(ALF*ANU)
ISN 0907             RE=PER*(DPDX-GTH*ROF(APO,T1))*(R2-R1)/(ETAF(APO,TB)*ANUO)
ISN 0908             RNUS=AKHC/AKO
ISN 0909             AKHC=(AKEFC+AKEFH)/2.
              C
ISN 0910             WRITE(6,260)
ISN 0911             WRITE(6,261)
ISN 0912             DO 150 I=1,II
ISN 0913             DO 151 J=1,JJ
ISN 0914             TEMER=TEND(I,J)-T(I,J)
ISN 0915             WRITE(6,262) I,J,SEND(I,J),TEND(I,J),TTUIM(I,J),TEMER
ISN 0916        151  CONTINUE
ISN 0917        150  CONTINUE
ISN 0918             WRITE(6,272)
ISN 0919             DO 152 I=1,II
ISN 0920             DO 153 J=1,JJ1
ISN 0921             WRITE(6,273) I,J,MVR(I,J),Q(I,J)
ISN 0922        153  CONTINUE
ISN 0923        152  CONTINUE
ISN 0924             WRITE(6,281)
ISN 0925             WRITE(6,282)
ISN 0926             DO 154 I=1,II1
ISN 0927             DO 155 J=1,JJ
ISN 0928             WRITE(6,283) I,J,MVTH(I,J)
ISN 0929        155  CONTINUE
ISN 0930        154  CONTINUE
ISN 0931             WRITE(6.285)
ISN 0932             DO 157 J=1,JJ1
ISN 0933             WRITE(6,286) J,QPH(J),QPC(J)
ISN 0934        157  CONTINUE
ISN 0935             WRITE(6,287)
ISN 0936             WRITE(6,288)
ISN 0937             DO 158 I=1,II
ISN 0938             WRITE(6,289) I,QT(I)
```

```
ISN 0939         158 CONTINUE
ISN 0940             WRITE(6,290) QPHT,QPCT,QTAV
ISN 0941             PR=ANU/ALF
ISN 0942             RNUS=AKHC/AKO
ISN 0943             WRITE(6,291) ALF,ANU,AKO,PR,RA,RE   ,RNUS
ISN 0944             WRITE(6,292)
ISN 0945             WRITE(6,293) AKAV,AKEFH,AKEFC,AKHC
ISN 0946         200 FORMAT('1',20X,'SR',5X,'KOUNT=',1X,I3,///)
ISN 0947         201 FORMAT('0',5X,10F11.3)
ISN 0948         210 FORMAT('0',20X,'TR TEMPERATURAS',///)
ISN 0949         211 FORMAT('0',5X,10F11.3)
ISN 0950         218 FORMAT('0',5X,I4)
ISN 0951         260 FORMAT('1',30X,'RESULTS FROM THE LAST ITIRATION',/)
ISN 0952         261 FORMAT('0',' I    J    STREAM FUNCTION    TEMPERATURES     DIMENSIONLE
                    1SS TEMPERATURES,K    TEMER')
ISN 0953         262 FORMAT('0',I2,2X,I2,5X,F10.3,8X,F8.3,15X,F10.3,10X,F5.2)
ISN 0954         272 FORMAT('1',' I    J      MVR (KG/(S*M**2)    Q (W)')
ISN 0955         273 FORMAT('0',I2,2X,I2,8X,F10.4,6X,F10.3)
ISN 0956         281 FORMAT('1','MVTH,COMPONENT OF MASS VELOCITY PERPENDICULAR TO MVR')
ISN 0957         282 FORMAT('0',' I    J      MVTH(KG/(S*M**2)')
ISN 0958         283 FORMAT('0',I2,2X,I2,8X,F10.3)
ISN 0959         285 FORMAT('1',' J        QPH(W)          QPC(W)')
ISN 0960         286 FORMAT('0',I2,4X,F10.3,4X,E10.3)
ISN 0961         287 FORMAT('1','HEAT FLOW THROUGH THE INSULATION,QT,QPHT,QPCT ,QTAV')
ISN 0962         288 FORMAT('0',' I       QT(W)')
ISN 0963         289 FORMAT('0',I2,4X,E10.3)
ISN 0964         290 FORMAT('0',' QPHT=',E10.3,'QPCT=',F10.3,'QTAV=',E10.3,///)
ISN 0965         291 FORMAT('0','ALF=',E11.4,'ANU=',E11.4,'AKO=',E11.4,'PR=',E11.4,'RA=
                    1',E11.4,'RE=',F11.4,5X,'RNUS=',E11.4,//)
ISN 0966         292 FORMAT('0','EFFECTIVE THERMAL CONDUCTIVITY, W/(K*M)',//)
ISN 0967         293 FORMAT('0','AKAV=',E11.4,'AKEFH=',F11.4,'AKEFH=',E11.4,'AKHC='
                    1,E11.4)
ISN 0968         300 FORMAT(5X,7I5,5X,F5.2)
ISN 0969         301 FORMAT(5X,3F10.5)
ISN 0970         302 FORMAT(5X,3F10.3)
ISN 0971         303 FORMAT(5X,F5.2,2E10.3)
ISN 0972         304 FORMAT(5X,3E10.3)
ISN 0973         305 FORMAT(5X,2F10.4)
ISN 0974             STOP
ISN 0975             END

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=00,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =      974 ,PROGRAM SIZE =   203798

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******
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LEVEL 21.7 ( JAN 73 )                    OS/360  FORTRAN H

COMPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=60,SIZE=0000K,
                   SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF
C                                                                       GELB  10
C     ..................................................................GELB  20
C                                                                       GELB  30
C        SUBROUTINE GELB                                                GELB  40
C                                                                       GELB  50
C        PURPOSE                                                        GELB  60
C           TO SOLVE A SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS WITH A   GELB  70
C           COEFFICIENT MATRIX OF BAND STRUCTURE.                       GELB  80
C                                                                       GELB  90
C        USAGE                                                          GELB 100
C           CALL GELB(R,A,M,N,MUD,MLD,EPS,IER)                          GELB 110
C        DESCRIPTION OF PARAMETERS                                      GELB 130
C           R      - M BY N RIGHT HAND SIDE MATRIX (DESTROYED).         GELB 140
C                    ON RETURN R CONTAINS THE SOLUTION OF THE EQUATIONS.GELB 150
C           A      - M BY M COEFFICIENT MATRIX WITH BAND STRUCTURE      GELB 160
C                    (DESTROYED).                                       GELB 170
C           M      - THE NUMBER OF EQUATIONS IN THE SYSTEM.             GELB 180
C           N      - THE NUMBER OF RIGHT HAND SIDE VECTORS.             GELB 190
C           MUD    - THE NUMBER OF UPPER CODIAGONALS (THAT MEANS        GELB 200
C                    CODIAGONALS ABOVE MAIN DIAGONAL).                  GELB 210
C           MLD    - THE NUMBER OF LOWER CODIAGONALS (THAT MEANS        GELB 220
C                    CODIAGONALS BELOW MAIN DIAGONAL).                  GELB 230
C           EPS    - AN INPUT CONSTANT WHICH IS USED AS RELATIVE        GELB 240
C                    TOLERANCE FOR TEST ON LOSS OF SIGNIFICANCE.        GELB 250
C           IER    - RESULTING ERROR PARAMETER CODED AS FOLLOWS         GELB 260
C                    IER=0  - NO ERROR,                                 GELB 270
C                    IER=-1 - NO RESULT BECAUSE OF WRONG INPUT PARAME-  GELB 280
C                             TERS M,MUD,MLD OR BECAUSE OF PIVOT ELEMENTGELB 290
C                             AT ANY ELIMINATION STEP EQUAL TO 0,       GELB 300
C                    IER=K  - WARNING DUE TO POSSIBLE LOSS OF SIGNIFI-  GELB 310
C                             CANCE INDICATED AT ELIMINATION STEP K+1,  GELB 320
C                             WHERE PIVOT ELEMENT WAS LESS THAN OR      GELB 330
C                             EQUAL TO THE INTERNAL TOLERANCE EPS TIMES GELB 340
C                             ABSOLUTELY GREATEST ELEMENT OF MATRIX A.  GELB 350
C                                                                       GELB 360
C        REMARKS                                                        GELB 370
C           BAND MATRIX A IS ASSUMED TO BE STORED ROWWISE IN THE FIRST  GELB 380
C           ME SUCCESSIVE STORAGE LOCATIONS OF TOTALLY NEEDED MA        GELB 390
C           STORAGE LOCATIONS, WHERE                                    GELB 400
C             MA=M*MC-ML*(ML+1)/2    AND    ME=MA-MU*(MU+1)/2    WITH   GELB 410
C             MC=MIN(M,1+MUD+MLD),  ML=MC-1-MLD,  MU=MC-1-MUD.          GELB 420
C           RIGHT HAND SIDE MATRIX R IS ASSUMED TO BE STORED COLUMNWISE GELB 430
C           IN N*M SUCCESSIVE STORAGE LOCATIONS. ON RETURN SOLUTION     GELB 440
C           MATRIX R IS STORED COLUMNWISE TOO.                          GELB 450
C           INPUT PARAMETERS M, MUD, MLD SHOULD SATISFY THE FOLLOWING   GELB 460
C           RESTRICTIONS     MUD NOT LESS THAN ZERO                     GELB 470
C                            MLD NOT LESS THAN ZERO                     GELB 480
C                            MUD+MLD NOT GREATER THAN 2*M-2.            GELB 490
C           NO ACTION BESIDES ERROR MESSAGE IER=-1 TAKES PLACE IF THESE GELB 500
C           RESTRICTIONS ARE NOT SATISFIED.                             GELB 510
C           THE PROCEDURE GIVES RESULTS IF THE RESTRICTIONS ON INPUT    GELB 520
C           PARAMETERS ARE SATISFIED AND IF PIVOT ELEMENTS AT ALL       GELB 530
C           ELIMINATION STEPS ARE DIFFERENT FROM 0. HOWEVER WARNING     GELB 540
C           IER=K - IF GIVEN - INDICATES POSSIBLE LOSS OF SIGNIFICANCE. GELB 550
C           IN CASE OF A WELL SCALED MATRIX A AND APPROPRIATE TOLERANCE GELB 560
C           EPS, IER=K MAY BE INTERPRETED THAT MATRIX A HAS THE RANK K. GELB 570
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               C           NO WARNING IS GIVEN IF MATRIX A HAS NO LOWER CODIAGONAL.
               C
               C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
               C           NONE
               C
               C        METHOD
               C           SOLUTION IS DONE BY MEANS OF GAUSS ELIMINATION WITH
               C           COLUMN PIVOTING ONLY, IN ORDER TO PRESERVE BAND STRUCTURE
               C           IN REMAINING COEFFICIENT MATRICES.
               C
               C     .................................. .........................
               C
ISN 0002             SUBROUTINE GELB(R,A,M,N,MUD,MLD,EPS,IER)
               C
               C
ISN 0003             DIMENSION R(1),A(1)
ISN 0004             DOUBLE PRECISION R,A,PIV,TB,TOL
               C
               C     TEST ON WRONG INPUT PARAMETERS
ISN 0005             IF(MLD)47,1,1
ISN 0006           1 IF(MUD)47,2,2
ISN 0007           2 MC=1+MLD+MUD
ISN 0008             IF(MC+1-M-M)3,3,47
               C
               C     PREPARE INTEGER PARAMETERS
               C        MC=NUMBER OF COLUMNS IN MATRIX A
               C        MU=NUMBER OF ZEROS TO BE INSERTED IN FIRST ROW OF MATRIX A
               C        ML=NUMBER OF MISSING ELEMENTS IN LAST ROW OF MATRIX A
               C        MR=INDEX OF LAST ROW IN MATRIX A WITH MC ELEMENTS
               C        MZ=TOTAL NUMBER OF ZEROS TO BE INSERTED IN MATRIX A
               C        MA=TOTAL NUMBER OF STORAGE LOCATIONS NECESSARY FOR MATRIX A
               C        NM=NUMBER OF ELEMENTS IN MATRIX R
ISN 0009           3 IF(MC-M)5,5,4
ISN 0010           4 MC=M
ISN 0011           5 MU=MC-MUD-1
ISN 0012             ML=MC-MLD-1
ISN 0013             MR=M-ML
ISN 0014             MZ=(MU*(MU+1))/2
ISN 0015             MA=M*MC-(ML*(ML+1))/2
ISN 0016             NM=N*M
               C
               C     MOVE ELEMENTS BACKWARD AND SEARCH FOR ABSOLUTELY GREATEST ELEMENT
               C     (NOT NECESSARY IN CASE OF A MATRIX WITHOUT LOWER CODIAGONALS)
ISN 0017             IER=0
ISN 0018             PIV=0.
ISN 0019             IF(MLD)14,14,6
ISN 0020           6 JJ=MA
ISN 0021             J=MA-MZ
ISN 0022             KST=J
ISN 0023             DO 9 K=1,KST
ISN 0024             TB=A(J)
ISN 0025             A(JJ)=TB
ISN 0026             TB=DABS(TB)
ISN 0027             IF(TB-PIV)8,8,7
ISN 0028           7 PIV=TB
ISN 0029           8 J=J-1
ISN 0030           9 JJ=JJ-1
               C
```

```
              C      INSERT ZEROS IN FIRST MU ROWS (NOT NECESSARY IN CASE MZ=0)
ISN 0031             IF(MZ)14,14,10
ISN 0032          10 JJ=1
ISN 0033             J=1+MZ
ISN 0034             IC=1+MUD
ISN 0035             DO 13 I=1,MU
ISN 0036             DO 12 K=1,MC
ISN 0037             A(JJ)=0.
ISN 0038             IF(K-IC)11,11,12
ISN 0039          11 A(JJ)=A(J)
ISN 0040             J=J+1
ISN 0041          12 JJ=JJ+1
ISN 0042          13 IC=IC+1
              C
              C      GENERATE TEST VALUE FOR SINGULARITY
ISN 0043          14 TOL=EPS*PIV
              C
              C
              C      START DECOMPOSITION LOOP
ISN 0044             KST=1
ISN 0045             IDST=MC
ISN 0046             IC=MC-1
ISN 0047             DO 38 K=1,M
ISN 0048             IF(K-MR-1)16,16,15
ISN 0049          15 IDST=IDST-1
ISN 0050          16 ID=I[illegible]
ISN 0051             ILR=[illegible]
ISN 0052             IF([illegible])18,18,17
ISN 0053          17 IL[illegible]
ISN 0054          18 I[illegible]
              C
              C      [illegible] SEARCH IN FIRST COLUMN (ROW INDEXES FROM I=K UP TO I=ILR)
ISN 0055             [illegible]
ISN 0056             [illegible]=K,ILR
ISN 0057             [illegible](A(II))
ISN 0058             IF(TB-PIV)20,20,19
ISN 0059          19 PI[illegible]
ISN 0060             J[illegible]
ISN 0061             [illegible]
ISN 0062          20 I[illegible]I-MR)22,22,21
ISN 0063          21 [illegible]ID-1
ISN 0064          2[illegible]=II+ID
              C
              C      [illegible]ST ON SINGULARITY
ISN 0065             [illegible](PIV)47,47,23
ISN 0066          23 [illegible](IER)26,24,26
ISN 0067          24 [illegible](PIV-TOL)25,25,26
ISN 0068          25 [illegible]R=K-1
ISN 0069          26 [illegible]IV=1./A(JJ)
              C
              C      [illegible]VOT ROW REDUCTION AND ROW INTERCHANGE IN RIGHT HAND SIDE R
ISN 0070             [illegible]=J-K
ISN 0071             [illegible] 27 I=K,NM,M
ISN 0072             [illegible]I+ID
ISN 0073             TB=PIV*R(II)
ISN 0074             R(II)=R(I)
ISN 00[illegible]     27 R(I)=TB
              C
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```
                    PIVOT ROW REDUCTION AND ROW INTERCHANGE IN COEFFICIENT MATRIX A   GELB1730
ISN 0076                II=KST                                                     GELB1740
ISN 0077                J=JJ+IC                                                    GELB1750
ISN 0078                DO 28 I=JJ,J                                               GELB1760
ISN 0079                TB=PIV*A(I)                                                GELB1770
ISN 0080                A(I)=A(II)                                                 GELB1780
ISN 0081                A(II)=TB                                                   GELB1790
ISN 0082             28 II=II+1                                                    GELB1800
                C                                                                  GELB1810
                C       ELEMENT REDUCTION                                          GELB1820
ISN 0083                IF(K-ILR)29,34,34                                          GELB1830
ISN 0084             29 ID=KST                                                     GELB1840
ISN 0085                II=K+1                                                     GELB1850
ISN 0086                MU=KST+1                                                   GELB1860
ISN 0087                MZ=KST+IC                                                  GELB1870
ISN 0088                DO 33 I=II,ILR                                             GELB1880
                C                                                                  GELB1890
                C       IN MATRIX A                                                GELB1900
ISN 0089                ID=ID+MC                                                   GELB1910
ISN 0090                JJ=I-MR-1                                                  GELB1920
ISN 0091                IF(JJ)31,31,30                                             GELB1930
ISN 0092             30 ID=ID-JJ                                                   GELB1940
ISN 0093             31 PIV=-A(ID)                                                 GELB1950
ISN 0094                J=ID+1                                                     GELB1960
ISN 0095                DO 32 JJ=MU,MZ                                             GELB1970
ISN 0096                A(J-1)=A(J)+PIV*A(JJ)                                      GELB1980
ISN 0097             32 J=J+1                                                      GELB1990
ISN 0098                A(J-1)=0.                                                  GELB2000
                C                                                                  GELB2010
                C       IN MATRIX R                                                GELB2020
ISN 0099                J=K                                                        GELB2030
ISN 0100                DO 33 JJ=I,NM,M                                            GELB2040
ISN 0101                R(JJ)=R(JJ)+PIV*R(J)                                       GELB2050
ISN 0102             33 J=J+M                                                      GELB2060
ISN 0103             34 KST=KST+MC                                                 GELB2070
ISN 0104                IF(ILR-MR)36,35,35                                         GELB2080
ISN 0105             35 IC=IC-1                                                    GELB2090
ISN 0106             36 ID=K-MR                                                    GELB2100
ISN 0107                IF(ID)38,38,37                                             GELB2110
ISN 0108             37 KST=KST-ID                                                 GELB2120
ISN 0109             38 CONTINUE                                                   GELB2130
                C       END OF DECOMPOSITION LOOP                                  GELB2140
                C                                                                  GELB2150
                C                                                                  GELB2160
                C       BACK SUBSTITUTION                                          GELB2170
ISN 0110                IF(MC-1)46,46,39                                           GELB2180
ISN 0111             39 IC=2                                                       GELB2190
ISN 0112                KST=MA+ML-MC+2                                             GELB2200
ISN 0113                II=M                                                       GELB2210
ISN 0114                DO 45 I=2,M                                                GELB2220
ISN 0115                KST=KST-MC                                                 GELB2230
ISN 0116                II=II-1                                                    GELB2240
ISN 0117                J=II-MR                                                    GELB2250
ISN 0118                IF(J)41,41,40                                              GELB2260
ISN 0119             40 KST=KST+J                                                  GELB2270
ISN 0120             41 DO 43 J=II,NM,M                                            GELB2280
ISN 0121                TB=R(J)                                                    GELB2290
ISN 0122                MZ=KST+IC-2                                                GELB2300
```

```
   ISN 0123            ID=J                                                    GELB2310
   ISN 0124            DO 42 JJ=KST,MZ                                         GELB2320
   ISN 0125            ID=ID+1                                                 GELB2330
   ISN 0126         42 TH=TH-A(JJ)*R(ID)                                       GELB2340
   ISN 0127         43 R(J)=TH                                                 GELB2350
   ISN 0128            IF(IC-MC)44,45,45                                       GELB2360
   ISN 0129         44 IC=IC+1                                                 GELB2370
   ISN 0130         45 CONTINUE                                                GELB2380
   ISN 0131         46 RETURN                                                  GELB2390
                 C                                                             GELB2400
                 C                                                             GELB2410
                 C     ERROR RETURN                                            GELB2420
   ISN 0132         47 IER=-1                                                  GELB2430
   ISN 0133            RETURN                                                   GELB244
   ISN 0134            END                                                     GELB2450

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=00,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =      133 ,PROGRAM SIZE =      2786

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                          1?9K BYTES OF CORE NOT USED
```

```
LEVEL 21.7 ( JAN 73 )                              OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF
  ISN 0002              FUNCTION ROF(P,T)
                C       DENSITY OF CO 2.(P=ATM,T=K,RO=KG/M**3)
  ISN 0003              PO=1.
  ISN 0004              TO=273.16
  ISN 0005              IF(T-516.) 1, 1, 2
  ISN 0006            1 CK=.0134
  ISN 0007              GO TO 5
  ISN 0008            2 IF(T-750.) 3, 4, 4
  ISN 0009            3 CK=(650.-T)* 1.E-4
  ISN 0010              GO TO 5
  ISN 0011            4 CK=-.01
  ISN 0012            5 PF=P/PO
  ISN 0013              TF=TO/T
  ISN 0014              ROF=1.9635*PF*TF*(1.+CK*PF*(TF**5))
  ISN 0015              RETURN
                C       DENSITY OF CO 2.(P=ATM,T=K,RO=KG/M**3)
  ISN 0016              END

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=00,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*       SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =       15 ,PROGRAM SIZE =       659

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                   205K BYTES OF CORE NOT USED
```

```
LEVEL 21.7 ( JUN 73 )                                   OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF
   ISN 0002          FUNCTION CPF(P,T)
             C             SPECIFIC HEAT OF CO2 (P=ATM, T=K, CP= WS/(KG,K))
   ISN 0003          P0=1.
   ISN 0004          T0=273.16
   ISN 0005          PF=P/P0
   ISN 0006          TF=T0/T
   ISN 0007          IF(PF-1.) 1,1,2
   ISN 0008        1 FCP = PF -1.
   ISN 0009          GO TO 3
   ISN 0010        2 FCP = (PF -1.)**1.05
   ISN 0011        3 CP0=.119+3.51E-4*T-2.34E-7*T*T+5.00E-11*T*T*T
   ISN 0012          CPF = CP0*(1.+1.0[illegible]-2*FCP*(TF**3.35))          *4186.
   ISN 0013          RETURN
   ISN 0014          END

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=00,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =       13 ,PROGRAM SIZE =       524

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                205K BYTES OF CORE NOT USED
```

```
LEVEL 21.7 ( JAN 71 )                                  OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF
   ISN 0002             FUNCTION ETAF(P,T)
                C           DYNAMIC VISCOSITY OF CO2 (P=ATM, T=K, ETA=(KG/(M*S))
   ISN 0003             P0=1.
   ISN 0004             T0=273.16
   ISN 0005             PF=P/P0
   ISN 0006             TF=T0/T
   ISN 0007             ETA0=(1.56E-7*SQRT (T))/(1.+(229./T))
   ISN 0008             ETAF=ETA0*(1.+4.78E-3*(PF-1.)*((TF**3)))       *9.9068
   ISN 0009             RETURN
   ISN 0010             END

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=00,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =        9 ,PROGRAM SIZE =       424

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                  205K BYTES OF CORE NOT USED
```

```
LEVEL 21.7 ( JAN 73 )                                  OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=00,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF
  ISN 0002            FUNCTION CLAMF(P,T)
              C            THERMAL CONDUCTIVITY OF CO2 (P=ATM, T=K, L=W/(M*K)
  ISN 0003            P0=1.
  ISN 0004            T0=273.16
  ISN 0005            PF=P/P0
  ISN 0006            TF=T0/T
  ISN 0007            IF(PF-1.) 4,4,5
  ISN 0008          4 FCL = PF-1.
  ISN 0009            GO TO 6
  ISN 0010          5 FCL = (PF -1.)**1.25
  ISN 0011          6 IF(T-T0-725.) 1,1,2
  ISN 0012          1 CA=3.4943E2
  ISN 0013            CB=1.6768E5
  ISN 0014            CC=2.7311E7
  ISN 0015            GO TO 3
  ISN 0016          2 CA=4.0476E2
  ISN 0017            CC=-1.9206E7
  ISN 0018            CB=1.5704E5
  ISN 0019          3 CLAM0=(SQRT (T))/(CA+(CB/T)+(CC/(T*T)))
  ISN 0020            CLAMF=CLAM0*(1.+2.14E-3*FCL*(TF**2.36))      *1.1623
  ISN 0021            RETURN
  ISN 0022            END

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=00,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,NOID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =      21 ,PROGRAM SIZE =      674

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                205K BYTES OF CORE NOT USED

*STATISTICS*  NO  DIAGNOSTICS THIS STEP
```

RESUMO

Uma solução numerica foi obtida para as distribuições de fluxo e de temperatura em isolamentos térmicos do tipo fibra. Foram consideradas geometrias retangular e cilíndrica. As condições de contorno utilizadas também incluíram os casos de parede quente permeável. Para cavidades retangulares foi obtida uma boa concordância com resultados publicados, tanto numéricos como experimentais. As distribuições de velocidade e de temperatura calculadas propiciaram um melhor entendimento dos fenomenos de escoamento e de transferência de calor em isolamentos tipo fibra. Os números de Nusselt, locais e médio, obtidos para a parede fria constituem informações úteis para o projeto de isolações do tipo de fibras para vasos e tubulações de usinas nucleares resfriadas a gás. O número de Nusselt médio foi correlacionado, com o Número de Rayleigh nos casos apenas de convecção natural, e com os números de Rayleigh e Reynolds nos casos de uma combinação de convecção natural com convecção forçada.

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