



## Variance reduction technique in a beta radiation beam using an extrapolation chamber



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### ABSTRACT

This paper aims to show how the variance reduction technique “Geometry splitting/Russian roulette” improves the statistical error and reduces uncertainties in the determination of the absorbed dose rate in tissue using an extrapolation chamber for beta radiation. The results show that the use of this technique can increase the number of events in the chamber cavity leading to a closer approximation of simulation result with the physical problem. There was a good agreement among the experimental measurements, the certificate of manufacture and the simulation results of the absorbed dose rate values and uncertainties. The absorbed dose rate variation coefficient using the variance reduction technique “Geometry splitting/Russian roulette” was 2.85%.

### 1. Introduction

The extrapolation chamber is an ionization chamber with two parallel electrodes. One of them is a circular collecting electrode surrounded by a guard ring separated by an insulating material. The other one is both the high voltage electrode and the entrance window for chamber operation. To modify the air mass of the sensitive volume, the distance between the electrodes (chamber depth) must be varied (ICRU, 1997).

The extrapolation chamber has been chosen as a primary instrument established for measuring beta radiation (Böhm, 1986; Caldas, 1986; Dias and Caldas, 1999; NIST, 2010; Bakshi et al., 2013; Vahabi et al., 2014). This chamber is supported by the Bragg-Gray theory. It determines absolutely the absorbed dose or the absorbed dose rate of beta radionuclides and other small penetration radiation sources at different depths. Thus, the main requirements of the cavity theory, which are small collecting surface and small air volume, are satisfied (Caldas, 1980; Oliveira and Caldas, 2005).

The commercial extrapolation chamber PTW model 23392, from Germany is recommended for absolute measurements of beta radiation and low energy X-rays absorbed doses (Böhm, 1986).

The development of complex electronic circuits, the introduction of multiple processor computers, parallelization algorithms and computational clusters have taken a step forward for particle transport Monte Carlo methods. The calculation times have decreased significantly. Therefore, it is possible to carry out simulations with geometries

increasingly similar to the actual situation of the specific problem. Moreover, Monte Carlo techniques are being widely used because of the chances of achieving simulations with powerful codes such as MCNP, EGSnrc, BEAMnrc, PENELOPE, ITS, ETRAN and GEANT (Zoubair et al., 2012; Saidi et al., 2013).

The radiation transport Monte Carlo method is used in many fields and applications such as radiation dosimetry, medical physics, radiation protection, shielding calculation, nuclear engineering, etc. (MNCP, 2008; Saidi et al., 2013).

Some extrapolation chambers have been successfully simulated by Monte Carlo method. These simulations have led to improvement in the knowledge of the physical factors used in the experimental measurements. In addition, in some studies, the transmission factors and the absorbed dose rates were calculated. Subsequently these parameters were compared with experimental data (Selvam et al., 2005; Neves et al., 2012; Behrens, 2013; Vahabi et al., 2014; Faria et al., 2015).

In the simulation of a radiation transport problem using the Monte Carlo method, it is necessary to consider if all possible random paths that contribute to the response are suitably sampled (Gualdrini and Ferrari, 2011). The regions in the geometry of the physical problem that achieve relatively a small number of interactions in a real situation, may also achieve a small number of interactions in the simulation, leading to a poor statistics for the Monte Carlo method in these regions (Van Wijk et al., 2011). One way to improve the number of events and the precision in these areas is to increase the computational simulation time, that is, to increase the number of initial particles. However, the

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use of variance reduction techniques (VRTs) have proven to be a much more effective solution in these cases.

VRTs have been used for a long time in particle transport Monte Carlo methods. There are methods for decreasing the variance (statistical error) in the estimated solution. Therefore, it is possible to reduce the computational time for Monte Carlo simulation (Booth, 1985; Lux and Koblinger, 1991). There are several VRTs, but the actual physical problems do not need the application of all these techniques to solve the concerned problem. The VRT Geometry splitting/Russian roulette is one of the most used techniques by Monte Carlo methods. Its objective is to sample more time in important regions (cells) and to sample less time in regions of minor importance (Booth, 1985). The most important region in an extrapolation chamber for Monte Carlo calculation is the sensitive volume (cavity).

The purpose of this paper is to show how the use of VRT Geometry splitting/Russian roulette can improve the statistical error and reduce uncertainties in the determination of the absorbed dose rates in tissue using an extrapolation chamber.

## 2. Materials and methods

### 2.1. Extrapolation chamber PTW model 23392

A commercial extrapolation chamber PTW model 23392 of the Laboratory for Calibration of Instruments (LCI) at the IPEN/CNEN/São Paulo, Brazil, was used (Fig. 1). This chamber was developed by Böhm (1986) at the Physikalisch-Technische Bundesanstalt (PTB), which is the German primary standard laboratory. The basic components of this chamber are: collecting electrode, guard ring, movable piston, micrometer screw, acrylic housing and the entrance window. This entrance window is very thin, so that the spectrum of incident particles will almost not be disturbed to cross it. The entrance window, the piston and the chamber internal structures are made of materials whose properties are equivalent to tissue, when considering mass stopping power of electron radiation. An air mass composes the sensitive volume of the chamber.

### 2.2. MCNP Monte Carlo transport code and VRT Geometry splitting/Russian roulette

MCNP is a general-purpose, continuous-energy, generalized-geometry, time-dependent, coupled neutron/photon/electron Monte Carlo transport code. In particular, the version 5 of MCNP (MCNP, 2008) can be used for the transport of neutrons, photons and electrons and for the coupled transport of neutron / photon / electron radiation. The user can instruct MCNP to make various tallies related to particle current, particle flux and energy deposition. In addition, the user may request to MCNP a report with these results in the output file.

The VRT Geometry splitting/Russian roulette divides the geometry into cells and assigns an importance to these cells. When the particle moves from cells of importance  $I_n$  to more importance cells  $I_m$ , ( $I_n < I_m$ ), it plays Splitting. That is, the particle is split into  $n = I_m / I_n$  particles with equal weight  $w = I_n$ . In case  $n$  is not an integer, the number of particles is increased to obtain a better sample and the weight of the

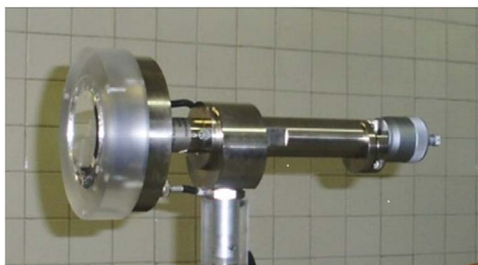


Fig. 1. Extrapolation chamber PTW model 23392 of the LCI/IPEN.

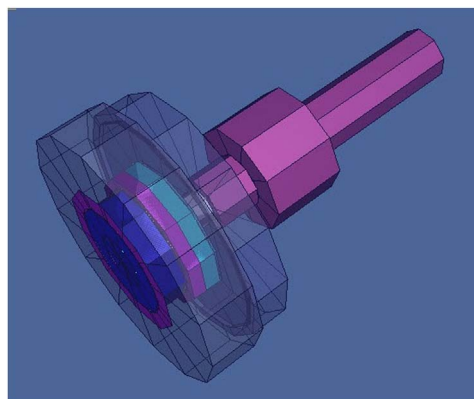


Fig. 2. Monte Carlo model of extrapolation chamber.

Table 1 Interactions in the simulations without and with VRT.

Interactions in the cavity	Without VRT	With VRT
	Photons	
Track entering	3838	30896
Population	3657	30710
Collisions	3	9
Number weighted energy	$8.03 \times 10^{-2}$	$8.06 \times 10^{-2}$
Flux weighted energy	$8.03 \times 10^{-2}$	$8.06 \times 10^{-2}$
Average track weight (relative)	1.00	1.00
Average track mfp (cm)	$3.40 \times 10^3$	$3.43 \times 10^3$
Interactions in the cavity	Without VRT	With VRT
Electrons		
Track entering	36149	291517
Population	35214	295101
Substeps	32244	251343
Number weighted energy	$5.56 \times 10^{-1}$	$5.58 \times 10^{-1}$
Flux weighted energy	$6.77 \times 10^{-1}$	$6.77 \times 10^{-1}$
Average track weight (relative)	1.00	1.00
Average track mean free path (cm)	9.36	9.37

particle is divided. When the particle moves to less important cells, it plays Russian roulette. The particle must move with probability  $I_m = I_n$  and weight  $w * I_n = I_m$ , or it must be killed with probability  $1 - (I_m = I_n)$  to avoid having to spend time on them. Splitting increases the calculation time and decreases the variance, whereas Russian roulette does the complete opposite. (Saidi et al., 2013).

The VRT Geometry splitting/Russian roulette does not introduce variance in the weight of the particle within a cell. The history of the variance is determined by the variation in the number of track scoring rather than a variation in particle weight (Booth, 1985).

The VRTs may vary from one study to another. In the simulation of this work, each electron was split into two electrons whenever it crossed a splitting plane 15 cm under the radiation source. Later, an electron was split into four electrons when it crossed a splitting plane 15 cm in front of the extrapolation chamber entrance window. The electrons crossing the planes corresponding to the entrance window, the cavity, the guard ring and acrylic housing cells were split into eight electrons.

### 2.3. Monte Carlo extrapolation chamber model

MCNP code has a flexibility in the design of a complex geometry. In the simulation of chamber geometry, the actual dimensions were used, taking into account measurements performed at the LCI (Antonio et al., 2014) and the characteristics listed in the manufacturer manual (PTW, 2002). The material densities used in the simulation for Mylar, acrylic, aluminum, steel, silver and air are 1.38, 1.19, 2.85, 8.06, 10.5, and  $1.205 \times 10^{-3} \text{ g/cm}^3$ , respectively (ICRU, 1997). To illustrate the

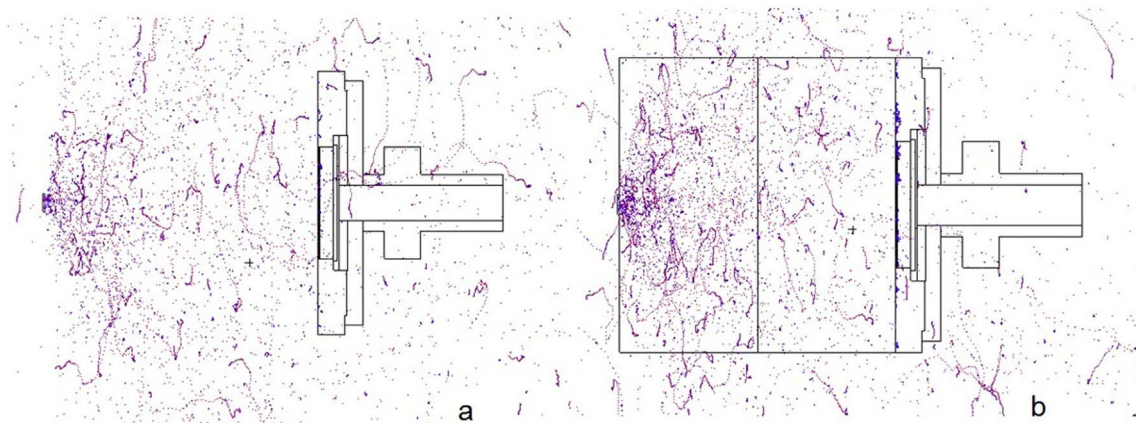


Fig. 3. Particle distribution simulation: a) without VRT; b) with VRT.

Table 2  
Comparison of the absorbed dose rate values and their uncertainties.

Data	Absorbed dose rate in tissue (μGy/s)
Without VRT	34.0 ± 4.3
With VRT	34.2 ± 0.9
Experimental Certificate	33.0 ± 1.8
	34.7 ± 0.6

MCNP Monte Carlo model of the extrapolation chamber the Vised version X\_22 S visualization tool was used (Fig. 2).

The null depth of an extrapolation chamber is the minimum distance between the electrodes to avoid them to touch each other, which could result in damage or rupture of the entrance window (Böhm, 1986; Caldas, 1986). The null depth is determined from the extrapolation of the ionization current for the positive and negative polarities of voltage in function of the chamber depth. The meeting point due to the extrapolation of the two lines is the null chamber depth. For the simulation, a 0.1184 mm null depth was considered (Antonio et al., 2014).

The Beta Secondary Standard, BSS1, <sup>90</sup>Sr/<sup>90</sup>Y radiation source of the LCI/IPEN was utilized in the simulation. Its nominal activity was 1850 MBq (1981) and the source-detector distance considered was 30 cm. The actual dimensions of the source, including its shielding, were considered. The <sup>90</sup>Sr/<sup>90</sup>Y source is distributed in the form of carbonate in a silver substrate. The energy spectrum used in the simulation was taken based on the ICRU Report No.56 (ICRU, 1997).

#### 2.4. Determination of the absorbed dose rate in tissue

For the absorbed dose rate determination, the \* f8 energy deposition MCNP tally was used. The dose rate calculated through the simulation was compared with the experimentally determined dose rate and with the dose rate provided by the source calibration certificate (PTW, 1981). The absorbed dose rate per gram in the tissue was calculated according to (MIT, 2016) by the following expression:

$$\dot{D} = A * \bar{E} * \left(\frac{\text{MeV}}{\text{g} \cdot \text{s}}\right) * 1, 60 * 10^{-13} * \left(\frac{\text{J}}{\text{MeV}}\right) * 10^3 \left(\frac{\text{g}}{\text{kg}}\right) = 1, 60 * 10^{-10} * A * \bar{E} \left(\frac{\text{Gy}}{\text{s}}\right) \tag{1}$$

where *A* is the source activity in Bq/g and  $\bar{E}$  is the mean energy in MeV per disintegration.

According to ISO (2004), the absorbed dose rate in the tissue within the sensitive volume of the extrapolation chamber was calculated by the expression:

$$\dot{D} = \dot{D} * s_{t,a}$$

where *s<sub>t,a</sub>* = 1.110 is the ratio of the stopping powers of tissue and air.

### 3. Results and discussion

In the simulations, the radiation transport was calculated according to the individual histories of electrons and photons that passed through all the material. All the electron interactions were considered. The ITS mode (DBCN 18 card = 1) for electron transport (MCNP, 2008) was utilized. The number of initial particles (nps) for the simulations was 50 × 10<sup>6</sup>, and it has fulfilled all MCNP code 10 statistical tests. Table 1 shows a comparison between the interactions occurred in the simulations without and with the use of VRT.

Fig. 3 shows a particle distribution simulation from the radiation source to the extrapolation chamber. For display purposes, only the distribution of 1000 particles can be observed. Fig. 3a shows the particle distribution without VRT, and Fig. 3b shows this distribution with VRT. Both figures show the particles distributed in air and in the fundamental components of the extrapolation chamber. Fig. 3b shows also the splitting plane for the VRT implementation.

It can be seen that with the application of the VRT Geometry splitting/Russian roulette, the possibility that events reach the cell corresponding to the cavity increases. Table 1 presents some comparison parameters of Monte Carlo simulation output files (see the parameters “track entering”, “population” and “substeps”). These parameters demonstrated an increase in the number of events in the region of the chamber cavity.

Monte Carlo simulation results show that the energy deposited by the source in the sensitive volume of the extrapolation chamber without applying the VRT was 2.29 × 10<sup>-7</sup> MeV. The associated relative error was 0.06. On the other hand, the energy deposited for the source in the sensitive volume considering VRT, was 2.30 × 10<sup>-7</sup> MeV and, the relative error was 0.014.

The absorbed dose rate per gram in the tissue was calculated by the expression (1). Table 2 shows the simulation results of the absorbed dose rates in tissue without applying the VRT and considering it. The absorbed dose rates are compared with the dose rate calculated experimentally and with the dose rate provided by the PTB calibration certificate (PTW, 1981). The calculated uncertainties are also shown in Table 2.

The simulation result without the use of VRT shows a relative error greater than 6%. The uncertainty calculated based on this relative error is high and the calculation variation coefficient was 12.7%. It would be necessary to increase the amount of initial particles (nps) to achieve a lower relative error, which would increase dramatically the computational time. The simulation result considering the use of the VRT shows a relative error of 1.4% and a low uncertainty. The variation coefficient was 2.85%. The difference between the experimental value and the calculated values without and with VRT were 2.9% and 3.5%, respectively.

The simulation result considering the use of the VRT is consistent, and improved the statistical result. The result and its uncertainty are in agreement with the experimental and calibration certificate absorbed dose rate values.

#### 4. Conclusions

The VRT is a powerful tool to improve the statistical results in the calculation of beta absorbed dose rates using an extrapolation chamber.

The use of VRT can increase the number of events in the chamber cavity leading to a closer approximation of the simulation result with the physical problem. A good agreement among the experimental measurements, the PTB certificate and the absorbed dose rate in tissue simulation for a source-detector distance of 30 cm has been obtained.

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